Exercises on Thermal Analysis

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This memo lists several exercises in thermal analysis as background for more detailed analysis of LCI and its components. For each exercise list any simplifying assumptions made.

Exercise 1

Consider a sphere at **1** AU from the sun. What is the steady state temperature if

(a) $\alpha = \epsilon = 1$?
(b) $\alpha = \epsilon = 0.05$?
(c) $\alpha = 0.05$ and $\epsilon = 0.1$?
(d) $\alpha = 0.1$ and $\epsilon = 0.05$?

Exercise 2

Consider a **50** kg polished aluminum sphere at **1** AU from the sun.

(a) What is the steady state temperature?
(b) What is the steady state temperature if there is an internal heater dissipating **50** W uniformly throughout the sphere?

How are these answers affected (changed) if the aluminum is anodized?

Exercise 3

Consider an object consisting of a **25** kg polished aluminum sphere inside a **25** kg polished aluminum spherical shell; the diameter of the shell is twice that of the sphere. The object is located at **1** AU from the sun.

Remark: ***Ignore how the sphere is held inside the shell.***

(a) What is the steady state temperature of the sphere? of the shell?
(b) If **50** W of power is dissipated uniformly in the sphere, what is the steady state temperature of the sphere? of the shell?

Remark: ***This is the typical* 1 *W/kg for spacecraft instruments.***

How are these answers affected (changed) if the shell is made of polished magnesium?

Exercise 4

Consider the object described in Exercise 3 with **50** W of power dissipated uniformly in the sphere but now located in low Earth orbit LEO.

Remark: ***Ignore how the sphere is held inside the shell.***

What is the steady state temperature of the sphere and of the shell if the object is located

(a) at noon, i.e., centered on the sunlit hemisphere?
(b) at midnight, i.e., in full eclipse?

Assuming the object enters the eclipse abruptly, plot the temperature of the sphere as a function of time (from entering the eclipse zone).

Exercise 5

Consider the object from Exercise 4 but now specify the suspension of the sphere in the shell as a tetrahedral space frame made from copper with the diameter of each strut equal to one-twentieth of the diameter of the sphere. The object is still in LEO with **50** W of power dissipated uniformly in the sphere.

What is the steady state temperature of the sphere and of the shell if the object is located

(a) at noon, i.e., centered on the sunlit hemisphere?
(b) at midnight, i.e., in full eclipse?

Assuming the object exits the eclipse abruptly, plot the temperature of the sphere as a function of time (from exiting the eclipse zone).

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1) Intensity of light from blackbody surface out into half-sphere $I_{bb}(\nu,T) = \frac{2h\nu^3}{c^2}\left(\frac{1}{e^{h\nu/kT}-1}\right)$

Stefan-Boltzmann law gives power per unit area of emitting body

$$\frac{P}{A} = \int_0^\infty I(\nu,T)\,d\nu \int_{\Omega} d\Omega = \pi\int_0^\infty I(\nu,T)\,d\nu = \frac{2\pi h}{c^2}\int_0^\infty \frac{\nu^3 d\nu}{e^{h\nu/kT}-1} = \frac{2\pi h}{c^2}\left(\frac{kT}{h}\right)^4\int_0^\infty \frac{u^3}{e^u-1}\,du$$

$$\int_0^\infty \frac{u^3\,du}{e^u-1} = \frac{\pi^4}{15}$$

$$\frac{P}{A} = \sigma T^4 \qquad \sigma = \frac{2\pi^5 k^4}{15c^2h^3} = 5.67\cdot 10^{-8}\ W/m^2K^4$$

for the sun $T_s \sim 5800K$ $\quad \frac{P}{A} = 6.42\cdot 10^7 \frac{W}{m^2}$

the radius of the sun $R_s \approx 0.7\cdot 10^9 m$

$$P_s = 4\pi R_s^2 \frac{P}{A} = 4\pi R_s^2 \sigma T_s^4$$

available at earth - divide by $4\pi R_{SE}^2$ $\quad R_{SE} = 1.5\cdot 10^{11} m$

The amount absorbed by a body at earth-sun distance depends on its ~~tempera~~ spectrally dependent absorption, which also depends on the temperature of the body and the cross-section of the body A_{object}

for a blackbody object

$$P_{absorbed} = 4\pi R_s^2 \sigma T_s^4 \left(\frac{A_{obj}}{4\pi R_{SE}^2}\right)$$

but for ~~and~~ object with $\alpha_{obj}(\lambda, T_{obj})$

$$P_{absorbed} = 4\pi R_s^2 \left(\frac{A_{obj}}{4\pi R_{SE}^2}\right)\frac{2h\pi}{c^2}\left(\int_0^\infty \frac{\alpha_{obj}(\nu, T_{obj})u^3 du}{e^u-1}\right)\left(\frac{kT_s}{h}\right)^4$$

because the integral is difficult to solve, the ~~abs~~ average absorptivity is measured for weighting in the solar spectrum and then pulled out of the integral

$$\alpha_{obj}(\text{heated by Sun}) = \frac{\int_0^\infty \alpha_{obj}(\nu, T_{obj})\, I_{sun}(\nu)\,d\nu}{\int_0^\infty I_{sun}(\nu)\,d\nu}$$

so that

$$P_{absorbed} = 4\pi R_s^2\left(\frac{A_{obj}}{4\pi R_{SE}^2}\right)\frac{2\pi h}{c^2}\left(\frac{kT_s}{h}\right)^4 \alpha_{obj}(\text{heated by Sun})\int_0^\infty \frac{u^3}{e^u-1}du$$

$$P_{absorbed} = \alpha_{obj}(\text{heated by Sun})\,\frac{R_s^2}{R_{SE}^2}\,A_{obj}\,\sigma T_s^4$$

In thermodynamic equilibrium $P_{absorbed} = P_{emitted}$

By Kirchoff's Law $\alpha(\nu) = \varepsilon(\nu)$ and $\alpha + \varepsilon + \rho = 1$

But the body is emitting over a different spectrum because it is a different temperature

average emissivity $\varepsilon = \dfrac{\int_0^\infty \varepsilon(\nu,T)\, E_b(\nu,T)\, d\nu}{\int_0^\infty E_b(\nu,T)\, d\nu}$

$$P_{emitted} = A_{obj}\, \pi \int_0^\infty \varepsilon(\nu, T_{obj})\, I_\nu\, d\nu = A_{obj}\, \pi\, \varepsilon \int_0^\infty I_\nu(\nu,T)\, d\nu = A_{obj}\, \varepsilon(T_{obj})\, \sigma T_{obj}^4$$

$$P_{emitted} = P_{absorbed}$$

$$A'_{obj}\, \varepsilon(T_{obj})\, \sigma T_{obj}^4 = \alpha_{obj}(\text{in sun region}) \frac{R_S^2}{R_{SE}^2} A_{obj}\, \sigma T_S^4$$

$$T_{obj}^4 = \frac{\alpha_{obj}}{\varepsilon_{obj}} \frac{R_S^2}{R_{SE}^2} T_S^4 \left(\frac{A_{obj}}{A'_{obj}}\right)$$ A'_{obj} = total surface area of object

$$T_{obj} = T_S \left(\frac{\alpha_{obj}(\text{solar region})}{\varepsilon_{obj}(\text{thermal region})}\right)^{1/4} \left(\frac{R_S}{R_{SE}}\right)^{1/2} \left(\frac{A_{obj}}{A'_{obj}}\right)^{1/4}$$ A_{obj} = cross-sectional area receiving sunlight

For a spherical body:

$$\left(\frac{A_{obj}}{A'_{obj}}\right)^{1/4} = \left(\frac{\pi R_{obj}^2}{4\pi R_{obj}^2}\right)^{1/4} = \frac{1}{\sqrt{2}}$$

$$\left(\frac{R_S}{R_{SE}}\right)^{1/2} = \left(\frac{.7 \cdot 10^9 m}{1.5 \cdot 10^{11} m}\right)^{1/2} \approx .0047$$

$$T_{obj} = T_S \left(\frac{\alpha_{obj}(\text{solar})}{\varepsilon_{obj}(\text{thermal})}\right)^{1/4} \left(\frac{.0047}{\sqrt{2}}\right)$$

$$T_S \approx 5800K$$

$$T_{obj} = \left(\frac{\alpha_{obj}(\text{solar})}{\varepsilon_{obj}(\text{thermal})}\right)^{1/4} (.048)\, T_S \approx \left(\frac{\alpha_{obj}(\text{solar})}{\varepsilon_{obj}(\text{thermal})}\right)^{1/4} 280K$$

	α	ε	α/ε	T_{obj}
a	1	1	1	280K
b	.05	.05	1	280K
c	.05	0.1	.5	235.5K
d	.1	.05	2	333K

2) A 50kg polished aluminum sphere 1AU from Sun (or anondized)

a) What is steady-state temperature?

for spherical object

$$T_{obj} = 280K\left(\frac{\alpha_{obj}(solar)}{\varepsilon_{obj}(thermal)}\right)^{1/4}$$

	α(solar)	ε(thermal)	α/ε	T_{obj}
polished aluminum	.09	.03	3.0	369K
anondized aluminum	.14	.84	.17	179K

b) 50W dissipated throughout sphere.

$P_{emitted} = -50W + A_{obj}\,\varepsilon(T_{obj})\,\sigma\,T_{obj}^4$ A = total surface area

~~$P_{absorbed} = \alpha_{obj}(solar)\frac{R_{S}^2}{R_{SE}^2}\left(\frac{A_{obj}}{A'_{obj}}\right)^{1/4}$~~

$P_{absorbed} = \alpha_{obj}(solar)\left(\frac{R_S}{R_{SE}}\right)^2 A'_{obj}\,\sigma T_S^4$ A' = cross-sectional area in sunlight

$P_{emitted} = P_{absorbed}$

$$-50W + A\varepsilon\sigma T^4 = \alpha\left(\frac{R_S}{R_{SE}}\right)^2 A'\sigma T_S^4$$

$$T^4 = \frac{\alpha\left(\frac{R_S}{R_{SE}}\right)^2 A'\sigma T_S^4 + 50W}{A\varepsilon\sigma}$$

$$T = \left[\frac{\alpha(R_S/R_{SE})^2 A'\sigma T_S^4 + 50W}{A\varepsilon\sigma}\right]^{1/4}$$

$\rho_{Al} \sim 2700\,kg/m^3$

$r = \left(\frac{3m}{4\pi\rho_{Al}}\right)^{1/3} = 0.164\,m$ $A' = .338\,m^2$ $A = .0846\,m^2$ $T_S = 5800K$

$$\therefore T = \left[\frac{\alpha}{\varepsilon}\left(6.16\cdot10^9 K^4\right) + \left(2.61\cdot10^9 K^4\right)\frac{1}{\varepsilon}\right]^{1/4}$$

	α	ε	α/ε	T_{obj}
polished Al	.09	.03	3	570K
anondized Al	.14	.84	.17	253K

3) Two concentric spheres (inner sphere, outer shell) each have mass = 25kg

inner sphere is labeled: 1, ρ_1, m_1

outer shell inner diameter: 2   outer diameter: 3 $r_1 < r_2 < r_3$

ρ_2, m_2 $2r_1 = r_3$

$R_{S} = 1AU = 1.5 \cdot 10^{11}\, m$

$\rho_{Al} = 2{,}700\, kg/m^3$ $\rho_{mg} = 1{,}738\, kg/m^3$ (Wiki)

get r_1, r_2, r_3

$m = \frac{4}{3}\pi(r_3^3 - r_2^3)\rho_{Al} = \frac{4}{3}\pi r_1^3 \rho_{Al}$

$r_3 = 2r_1$

$r_3^3 - r_2^3 = r_1^3$

$8r_1^3 - r_2^3 = r_1^3$

$r_2 = (7)^{1/3} r_1$

$r_1 = \left(\frac{3m}{4\pi\rho_{Al}}\right)^{1/3} = 0.13m$

$r_2 = 0.249m$

$r_3 = 0.261m$

$r_1 = \left(\frac{3m}{4\pi\rho_{Al}}\right)^{1/3} = 0.13m$

$m = \frac{4}{3}\pi(r_3^3 - r_2^3)\rho_{mg} = \frac{4}{3}\pi r_1^3 \rho_{Al}$

$r_3 = 2r_1$

$(8r_1^3 - r_2^3)\rho_{mg} = r_1^3 \rho_{Al}$

$8r_1^3\rho_{mg} - r_1^3\rho_{Al} = r_2^3\rho_{mg}$

$r_2^3 = \frac{(8\rho_{mg} - \rho_{Al})r_1^3}{\rho_{mg}}$

$r_2 = \left[\frac{(8\rho_{mg} - \rho_{Al})}{\rho_{mg}}\right]^{1/3} r_1 = 0.242m$

$r_3 = 0.261m$

Quoted in literature, from textbook by Siegel & Howell (1972)

$$\dot{Q}_{rad\,2\to1} = \frac{A_1\sigma(T_2^4 - T_1^4)}{\frac{1}{\varepsilon_1(T_1)} + \frac{A_1}{A_2}\left[\frac{1}{\varepsilon_2(T_2)} - 1\right]}$$

for steady state $\dot{Q}_{rad\,2\to1} = 0 \;\therefore\; T_2 = T_1$

Found before: $T_2 = 280K\left(\frac{\alpha_{obj}(solar)}{\varepsilon_{obj}(thermal)}\right)^{1/4}$

for polished Aluminum $\alpha(solar) \sim .09$ $\varepsilon(thermal) \sim .03$ ratio = 3

$T_2 = 368.5K = T_1$

for magnesium $.07 - 0.13 \sim \varepsilon(thermal)$ $\alpha(solar) \sim .09$ ratio = 1

$T_2 = 280K = T_1$

If 50W is dissipated uniformly from to sphere ...

$$\dot{Q}_{rad\,2\to1} = -50W = \frac{A_1\sigma(T_2^4 - T_1^4)}{\frac{1}{\varepsilon(T_1)} + \frac{A_1}{A_2}\left[\frac{1}{\varepsilon_2(T_2)} - 1\right]}$$

inner temperature is related to outer by:

$$T_1^4 = \frac{+50W\left\{\frac{1}{\varepsilon_1(T_1)} + \frac{A_1}{A_2}\left[\frac{1}{\varepsilon_2(T_2)} - 1\right]\right\}}{A_1\sigma} + T_2^4$$

$$T_1 = \left[\frac{50W\left\{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left[\frac{1}{\varepsilon_2} - 1\right]\right\}}{A_1\sigma} + T_2^4\right]^{1/4}$$

$\sigma = 5.67\cdot10^{-8}\ \frac{W}{m^2K^4}$

~~cases~~	case 1	case 2
ε_1	.03	.03
ε_2	.03	.09
r_1	0.13 m	0.13 m
$A_1 = 4\pi r_1^2$	.213 m^2	.213 m^2
r_2	0.249 m	0.242 m
$A_1/A_2 = (r_1/r_2)^2$	.2733	.2887
T_2	368.5K	280K
$\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left[\frac{1}{\varepsilon_2} - 1\right]$	42.2	36.3
$50/A_1\sigma$	$1.88\cdot10^8$	$1.88\cdot10^8$
T_1	403K	337K

with internal heater

4) $m_1 = 25\text{kg} = m_2 \qquad P_1 = 50\text{W}$

Just the shell...

for an object orbiting earth, the absorptivity is in thermal region

$$P_{abs} = \alpha_{obj}(\text{thermal})\left(\frac{R_E}{R_{EO}}\right)^2 A_o'' \sigma T_E^4$$

A_o'' = cross-sectional area illuminated by Earth

$$P_{emitted} = A_o\, \varepsilon(\text{thermal})\, \sigma T_o^4$$

A_o = total object area

$$P_{abs} = \alpha_{obj}(\text{solar})\left(\frac{R_S}{R_{SE}}\right)^2 A_o' \sigma T_S^4$$

~~P = power dissipated by the~~

A_o' = cross-sectional area illuminated by Sun.

$$\cancel{P_{abs\,total} = \alpha(\text{thermal})\left(\frac{R_E}{R_{EO}}\right)^2 A_o'' \sigma T_E^4 + \alpha(\text{solar})\left(\frac{R_S}{R_{SE}}\right)^2 A_o' \sigma T_S^4}$$

also have abosped reflected sunlight off earth 30% reflected

$$P_{abs\ reflected} = 0.3\, \alpha(\text{solar})\left(\frac{R_E}{R_{EO}}\right)^2 A_o'' \sigma T_S^4 \left(\frac{R_S}{R_{SE}}\right)^2$$

$$P_{absorbed} = \varepsilon(\text{thermal})\left(\frac{R_E}{R_{EO}}\right)^2 A'' \sigma T_E^4 + .3\alpha(\text{solar})\left(\frac{R_E}{R_{EO}}\right)^2\left(\frac{R_S}{R_{SE}}\right)^2 A'' \sigma T_S^4 + f_2\alpha(\text{solar})\left(\frac{R_S}{R_{SE}}\right)^2 A_o' \sigma T_S^4$$

$$P_{emitted} = \varepsilon(\text{thermal})\, \sigma T^4 A$$

$$P_{abs} = P_{emitted}$$

can consider as a factor $f_1 = .3$ at noon and $f_1 = 0$ at midnight

$f_2 = 1$ at noon, 0 at midnight

or $A' = 0$ at midnight

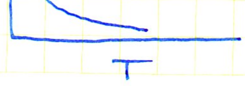
$$T_2 = \left[\frac{\varepsilon(th)\left(\frac{R_E}{R_{EO}}\right)^2 A'' \sigma T_E^4 + f_1\,\alpha(\text{solar})\left(\frac{R_E}{R_{EO}}\right)^2\left(\frac{R_S}{R_{SE}}\right)^2 A'' \sigma T_S^4 + \alpha(\text{solar})\left(\frac{R_S}{R_{SE}}\right)^2 A' \sigma T_S^4}{\varepsilon(th)\,\sigma A}\right]^{1/4}$$

$$T_2 = \left[\left(\frac{R_E}{R_{EO}}\right)^2\left(\frac{A''}{A}\right)T_E^4 + \frac{\alpha(\text{solar})}{\varepsilon(th)}\left(\frac{R_S}{R_{SE}}\right)^2 T_S^4\left[f_1\frac{A''}{A}\left(\frac{R_E}{R_{EO}}\right)^2 + \frac{A'}{A}\right]\right]^{1/4}$$

$T_E = 300\text{K} \qquad R_E/R_{EO} \sim 1/2$

	α_s	ε_2	α/ε	A''/A	A'/A	T_2	ε_1	T_1 using previous relation
noon Al	.09	.03	3	1/4	1/4	382 ~~467.4~~ K	.03	409 ~~483~~ K
noon Mg	.14	.84	.17	1/4	1/4	185 ~~148.49~~ K	.03	309 ~~300~~ K
midnight Al	.09	.03	3	1/4	0	197.4 K	.03	302 K
midnight Mg	.14	.84	.17	1/4	0	96 K	.03	299 K

suddenly



5) Tetrahedral frame connects them so $T_1 = T_2$

∴ is going to be the same as 2b)

~~polished~~ except now it is day/night + earth illuminated

$$T_2 = \left[\left(\frac{R_E}{R_{EO}}\right)^2 \left(\frac{A''}{A}\right) T_E^4 + \frac{\alpha(\text{solar})}{\epsilon(\text{thermal})}\left(\frac{R_S}{R_{SE}}\right)^2 T_S^4 \left[f_1 \frac{A''}{A}\left(\frac{R_E}{R_{EO}}\right)^2 + \frac{A'}{A}\right] + \frac{50W}{\epsilon(th)\sigma A}\right]^{1/4}$$

noon Al	382
noon Mg	~~[illegible]~~ 185
mid Al	197
mid Mg	96

Found on-line:

3.2 Heat transfer processes

3.2.1 Radiation

$\frac{A_1}{A_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Consider the radiative heat transfer between two concentric spheres as were shown in Figure 1. Assume that the surface of the inner sphere is at a uniform temperature T_1 and that the surface of the outer sphere is at a uniform temperature T_2. Let A_1 and ε_1 be the surface area and emissivity of the inner sphere, respectively, and A_2 and ε_2 be the surface area and emissivity of the outer sphere, respectively. Also assume that the surfaces of the spheres are diffuse-gray.

This problem has been presented as Example 8-3 in the textbook of Siegel and Howell (1972) to illustrate the use of the net radiation method for the solution of radiative heat transfer problems. The net radiative heat flow supplied to the smaller sphere is shown to be

$$\dot{Q}_{\mathrm{rad}} = \frac{A_1\sigma(T_2^4 - T_1^4)}{\dfrac{1}{\varepsilon_1(T_1)} + \dfrac{A_1}{A_2}\left[\dfrac{1}{\varepsilon_2(T_2)} - 1\right]} \qquad (1)$$

$\varepsilon_1 = \varepsilon_2 = \varepsilon$ $\quad \dfrac{A_1\sigma(T_2^4 - T_1^4)}{\frac{1}{\varepsilon}\left(1 + \frac{A_1}{A_2}\right) - \frac{A_1}{A_2}}$

$\varepsilon_2 \sigma T_2^4 + P =$

$\dot{Q}_{rad} + P_{out} = 0$

where σ is the Stefan-Boltzmann constant ($5.67 \cdot 10^{-8}$ W/(m^{2}K^4)). The notation has been chosen to emphasize the fact that the emissivities of both spheres may be functions of temperature. Here the sign has been chosen to be positive for the case where the inner sphere is a net receiver of energy ($T_2 > T_1$) and negative for the case where the inner sphere is a net supplier of energy ($T_2 < T_1$).

50J/s

Equation (1) is not very practical, since using it requires that we know the value of the emissivity of the furnace liner $\varepsilon_2(T_2)$. Usually we do not. However, if the size of the inner sphere is small when compared to the size of the outer sphere and if the emissivity of the outer sphere is not very small, Equation (1) simplifies to

$$\dot{Q}_{\mathrm{rad}} = \varepsilon_1(T_1)A_1\sigma(T_2^4 - T_1^4) \qquad (2)$$

which means that the net radiative heat flow is independent of the emissivity of the outer sphere. This result was also derived by Siegel and Howell.