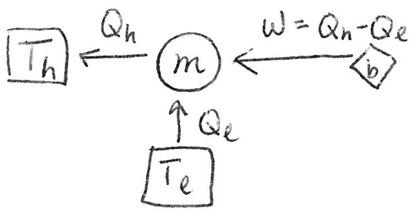
Meg Nodh

1. Heat Pump. (a) Show that for a reversible heat pump the energy required per unit of heat delivered inside the building is given by the Carnot efficiency $W/Q_h = \eta_c = \frac{T_h - T_e}{T_h}$

7/10



Reversible $\therefore$ $\Delta S_{TOT} = 0$

$S_h + S_e + S_m + S_b = 0 \qquad S_m = 0 \quad S_b = 0$

$$\frac{Q_h}{T_h} - \frac{Q_e}{T_e} = 0 \Rightarrow \frac{T_e}{T_h} = \frac{Q_e}{Q_h}$$

$$\frac{W}{Q_h} = \frac{Q_h - Q_e}{Q_h} = 1 - \frac{Q_e}{Q_h} = 1 - \frac{T_e}{T_h} = \frac{T_h - T_e}{T_h} = \frac{kT_h - kT_e}{kT_h} = \frac{\tau_h - \tau_e}{\tau_h}$$

What happens if the heat pump is not reversible? $\Delta S_{TOT} > 0$

$$\frac{Q_h}{T_h} - \frac{Q_e}{T_e} > 0 \Rightarrow \frac{Q_e}{Q_h} < \frac{T_e}{T_h} \Rightarrow \frac{Q_e}{Q_h} = 1 - \frac{W}{Q_h} < \frac{T_e}{T_h} \Rightarrow \frac{W}{Q_h} > 1 - \frac{T_e}{T_h}$$

more energy is required per unit of heat delivered inside the building

$$\frac{W}{Q_h} > \frac{T_h - T_e}{T_h}$$

(b) Assume that the electricity consumed by a reversible heat pump must itself be generated by a Carnot engine operating between τ_{hh} and τ_e. What is the ratio Q_{hh}/Q_h of the heat consumed at τ_{hh} to the heat delivered at τ_h?

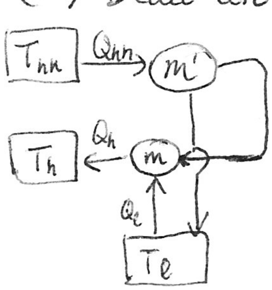
$$\left(\frac{W}{Q_{hh}}\right)_{rev} = \frac{T_{hh} - T_e}{T_{hh}} \qquad \left(\frac{W}{Q_h}\right)_{rev} = \frac{T_h - T_e}{T_h}$$

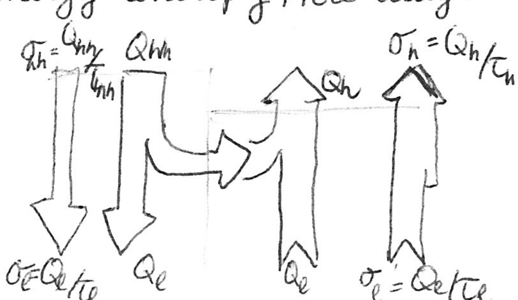
$$\frac{Q_{hh}}{Q_h} = \frac{\left(\frac{W}{Q_h}\right)_{rev}}{\left(\frac{W}{Q_{hh}}\right)_{rev}} = \frac{\left(\frac{T_h - T_e}{T_h}\right)}{\left(\frac{T_{hh} - T_e}{T_{hh}}\right)} = \frac{T_{hh}(T_h - T_e)}{T_h(T_{hh} - T_e)}$$

Give numerical values for $T_{hh} = 600K$, $T_h = 300K$, $T_e = 270K$.

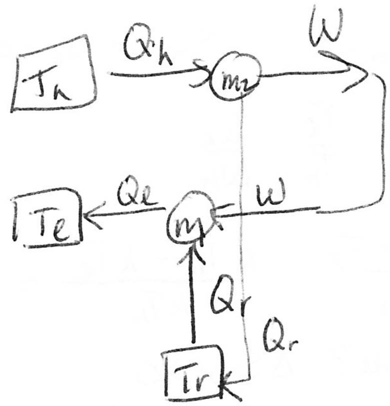
$$\frac{Q_{hh}}{Q_h} = \frac{600(300-270)}{300(600-270)} = \frac{2 \cdot 30}{330} = 0.1818$$

(c) Draw an energy-entropy flow diagram





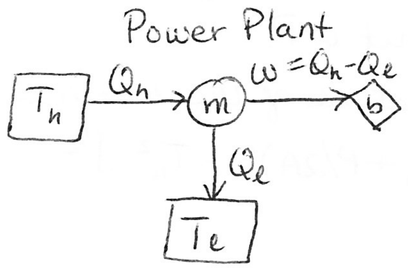
4. Heat Engine



Meg Noah

5. Thermal Pollution. A river with a water temperature $T_e = 20°C = 293K$ is to be used as the low temperature reservoir of a large power plant, with a steam temperature of $T_h = 500°C = 773K$. If ecological considerations limit the amount of heat that can be dumped into the river to 1500 MW, what is the largest electrical output that the plant can deliver?

$$\left(\frac{dQ_e}{dt}\right) \leq 1500 \, mw$$



$$\Delta S_{tot} = S_h + S_e + S_m + S_b = 0$$

$$\frac{-Q_h}{T_h} + \frac{Q_e}{T_e} = 0 \qquad \frac{Q_e}{Q_h} = \frac{T_e}{T_h}$$

$$W = Q_h - Q_e$$

$$\frac{dw}{dt} = \frac{dQ_h}{dt} - \frac{dQ_e}{dt} = \frac{T_h}{T_e}\frac{dQ_e}{dt} - \frac{dQ_e}{dt} = \frac{dQ_e}{dt}\left(\frac{T_h}{T_e} - 1\right)$$

$$\frac{dw}{dt} \leq (1500 \, mw)\left[\frac{773}{293} - 1\right] = 2747. \, mw$$

If improvements in hot-steam technology would permit raising T_h by 100°C, what effect would this have on the plant capacity

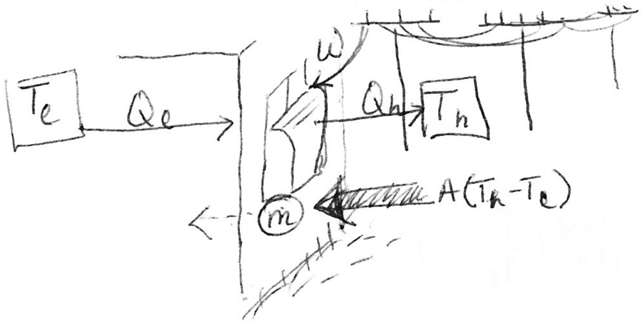
$$\frac{dw}{dt} \leq (1500 \, mw)\left[\frac{873}{293} - 1\right] = 2969 \, mw$$ increase production

3

6. Room Air Conditioner. A room air conditioner operates as a Carnot cycle refrigerator between an outside temperature T_h and a room at a lower temperature T_e. The room gains heat from the outdoors at a rate $A(T_h - T_e)$; this heat is removed by the air conditioner. The power supplied to the cooling unit is P.

(a) Show that the steady-state temperature of the room is

$$T_e = (T_h + P/2A) - [(T_h + P/2A)^2 - T_h^2]^{1/2}$$



$$\frac{dQ_e}{dt} = A(T_h - T_e) \qquad \frac{dW}{dt} = P$$

$$\Delta S_{TOT} = 0$$

$$-\frac{Q_e}{T_e} + \frac{Q_h}{T_h} = 0 \Rightarrow Q_h = \frac{T_h}{T_e} Q_e$$

$$Q_e + W = Q_h = \frac{T_h}{T_e} Q_e$$

$$\cancel{dQ} \qquad W = Q_e\left(\frac{T_h}{T_e} - 1\right)$$

$$\frac{dW}{dt} = \frac{dQ_e}{dt}\left(\frac{T_h}{T_e} - 1\right)$$

$$P = A(T_h - T_e)\left(\frac{T_h}{T_e} - 1\right)$$

$$(P/A)\, T_e = (T_h - T_e)(T_h - T_e)$$

$$T_h^2 - 2 T_e T_h + T_e^2 - (P/A) T_e = 0$$

$$T_e^2 + T_e(-P/A - 2T_h) + T_h^2 = 0$$

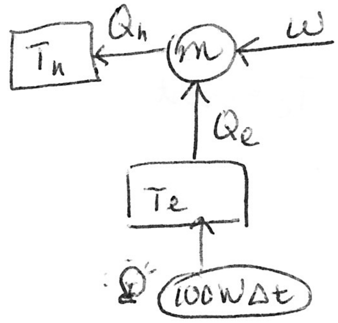
$$T_e = \frac{(P/A + 2T_h) \pm \sqrt{(P/A - 2T_h)^2 - 4T_h^2}}{2} = (T_h + P/2A) - \sqrt{(T_h + P/2A)^2 - T_h^2}$$

(b) If the outdoors is at 37°C ($\approx$ 307K) and the room is maintained at 17°C by a cooling power of 2kW, find the heat loss coef. A of the room in WK^{-1}. $T_h = 307K$ $T_e = 287K$ $P = 2000W$

$$A = \frac{P T_e}{(T_h - T_e)^2} = \frac{(2000W)\, 287K}{(307-287)^2\, K^2} = 1435\, WK^{-1}$$

Meg Noah

7. Light Bulb in a Refrigerator.



$$W = Q_h - Q_e$$

$$\Delta S_{TOT} \geq 0$$

$$\frac{Q_h}{T_h} + \frac{(100W\Delta t - Q_e)}{T_e} \geq 0 \qquad \cancel{P = \frac{dW}{dt} + \frac{dQ_h}{dt} + \frac{dQ_e}{dt}}$$

$$Q_h \geq \frac{T_h}{T_e}\left[Q_e - 100W\Delta t\right] \qquad \cancel{\frac{dQ_h}{dt} = P + \frac{dQ_e}{dt}}$$

$$\frac{dQ_h}{dt} \geq \frac{T_h}{T_e}\left[\frac{dQ_e}{dt} - 100W\right]$$

$$P + \frac{dQ_e}{dt} \geq \frac{T_h}{T_e}\left[\frac{dQ_e}{dt} - 100W\right] \qquad \cancel{P = 100W}$$

$$\gamma_c = \left(\frac{Q_e}{W}\right) = \frac{T_e}{T_h - T_e} \quad \text{for a refrigerator} \quad P = 100W$$

$$\frac{dQ_e}{dt} = \left(\frac{T_e}{T_h - T_e}\right)\frac{dW}{dt} = \left(\frac{T_e}{T_h - T_e}\right)P = \left(\frac{T_e}{T_h - T_e}\right)100$$

$$100\left[1 + \frac{T_e}{T_h - T_e}\right] \geq \frac{T_h}{T_e}\left[\left(\frac{T_e}{T_h - T_e}\right)100 + 100\right]$$

$$\therefore \quad T_e \geq T_h$$

So, no, it can't ~~heat~~ have $T_e < T_h$

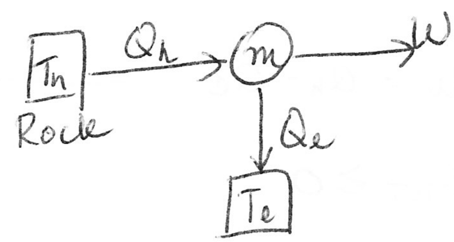
8. Geothermal Energy

$$đQ_h = -mc\,dT_h$$

$$dE = đQ + đW \qquad T_h = T_i \text{ to } T_h = T_f \qquad T_e = \text{cnst}$$

$$dW = -mc\,dT_h$$

$$= -mc(T_f - T_i)$$



$$c = 1\ J g^{-1} K^{-1}$$

$$T_i = 600°C$$

$$W = Q_h - Q_e$$

$$W = Q_h - \cancel{Q_e} = -mc(T_f - T_i) - Q_e$$

$$\cancel{\frac{Q_h}{T_h}} = \frac{Q_e}{T_e}$$

$$\int \frac{dQ_h}{T_h} = \int -\frac{mc\,dT_h}{T_n}$$

$$-mc \ln\frac{T_f}{T_i} = \frac{Q_e}{T_e}$$

$$S_h = -mc\ln\frac{T_f}{T_i}$$

$$S_e = \frac{Q_e}{T_e}$$

$$\boxed{W = -mc(T_f - T_i) + T_e c m \ln\frac{T_f}{T_i}}$$

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$c = 1\ J g^{-1} K^{-1}$ $\quad T_i = 873K$

$T_f = 383K$

$T_e = 293K$

$m = 10^{14}\ kg$

$$W = +(10^{14})(1)(490) + (293)(10^{14})(1)\ln\left(\frac{383}{873}\right)^{.43}$$

$$W = (618.5 \cdot 10^{14}\ J)\left(\frac{W}{s}\right)\frac{10^{-4}\ kW}{W}\left(\frac{3600s}{h}\right) = \cancel{0.1227 \cdot 10^{16}}\ 6.9 \cdot 10^{12}\ \frac{kW}{h}$$

$\cancel{4618}$