

10/10

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Statistical Physics
95,521
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Problem Set #1

1.1 Show that for a quasistatic adiabatic process in a perfect gas, with specific heats, $PV^\gamma = \text{const}$ where $\gamma = C_p/C_v$

$$dQ = 0 \quad (\text{adiabatic})$$

$$PV = RT \quad (1.5 \text{ equation for perfect gas})$$

$$dQ = dE + PdV = 0 \quad (1.17)$$

$$C_v dT + PdV = 0 \quad (1.18a)$$

using $P = \left(\frac{RT}{V}\right)$ from eq. of state for a perfect gas

$$C_v dT + \frac{RTdV}{V} = 0$$

using $C_p - C_v = R$ (1.20)

$$C_v dT + \left(\frac{C_p - C_v}{V}\right) T dV = 0$$

$$\frac{dT}{T} = \frac{dV}{V} \left(1 - \frac{C_p}{C_v}\right)$$

$$\frac{dT}{T} = (1 - \gamma) \frac{dV}{V}$$

$$\int \frac{dT}{T} = (1 - \gamma) \int \frac{dV}{V}$$

$$\ln T + \text{const} = (1 - \gamma) \ln V$$

$$\text{let const} = \ln C$$

$$\ln T + \ln C = (1 - \gamma) \ln V$$

$$\ln CT = \ln V^{(1-\gamma)}$$

$$CT = V^{(1-\gamma)}$$

$$\frac{CPV}{R} = V^{(1-\gamma)}$$

R

$$PV^\gamma = CR = \text{const}$$

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1.2 The molar energy of a monatomic gas which obeys Van der Waal's equation:

$$E = \frac{3}{2} RT - \frac{a}{V}$$

V = molar volume @ T a = const

$$V_1, T_1 \Rightarrow V_2, T_2 = ?$$

$$\delta Q = 0$$

adiabatic expansion: $\delta Q = 0$ $\Delta E = 0$

$$\Delta E = E_2 - E_1 = \left(\frac{3}{2} RT_2 - \frac{a}{V_2} \right) - \left(\frac{3}{2} RT_1 - \frac{a}{V_1} \right) = 0$$

$$\frac{3}{2} R(T_2 - T_1) = \frac{1}{a} \left(\frac{1}{V_2} - \frac{1}{V_1} \right)$$

$$T_2 = T_1 + \frac{2a}{3R} \left(\frac{1}{V_2} - \frac{1}{V_1} \right)$$

1.3 Calculate the work done against 1 mole of a perfect gas in an adiabatic quasistatic compression from V_1 to V_2 .

$$dQ = 0$$

$$dE = dW = -PdV$$

$$W = - \int_{V_1}^{V_2} PdV$$

$$C_v dT = -PdV \quad (1.18a)$$

$$W = \int_{T_1}^{T_2} C_v dT = C_v (T_2 - T_1) = \frac{C_v}{R} (P_2 V_2 - P_1 V_1)$$

$$\frac{R W}{C_v} = P_2 V_2 - P_1 V_1$$

$$\frac{(C_p - C_v)}{C_v} W = P_2 V_2 - P_1 V_1$$

$$(\gamma - 1) W = P_2 V_2 - P_1 V_1 \quad (2)$$

$$W = \frac{1}{(\gamma - 1)} (P_2 V_2 - P_1 V_1)$$

1.4 Enthalpy is $H \equiv E + PV$. Express heat capacity at $P = \text{constant}$ in terms of enthalpy.

$$dQ = dE + PdV = dH$$

$$dH = dE + PdV + \overset{P=\text{const}}{VdP} = dQ + \overset{P=\text{const}}{VdP} = dQ$$

$$C_p = \left(\frac{dQ}{dT} \right)_p = \left(\frac{\partial H}{\partial T} \right)_p$$

1.5 One mole of a perfect gas performs a quasistatic cycle which consists of the following four successive stages.

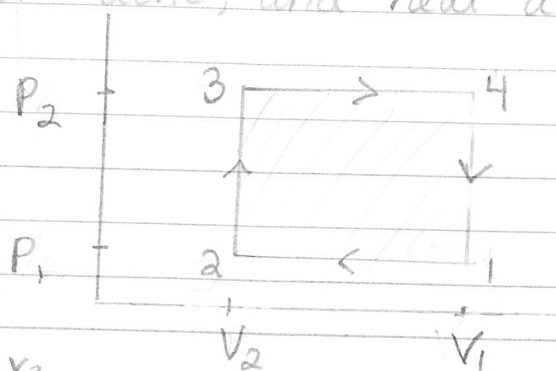
(i) from $(P_1, V_1) \rightarrow \Delta P = 0 \rightarrow (P_1, V_2)$

(ii) from $(P_1, V_2) \rightarrow \Delta V = 0 \rightarrow (P_2, V_2)$

(iii) from $(P_2, V_2) \rightarrow \Delta P = 0 \rightarrow (P_2, V_1)$

(iv) from $(P_2, V_1) \rightarrow \Delta V = 0 \rightarrow (P_1, V_1)$

find work done, and heat absorbed.



$$W_{12} = - \int_{V_1}^{V_2} P_1 dV = -P_1(V_2 - V_1)$$

$$W_{23} = - \int_{V_2}^{V_2} P dV = 0$$

$$W_{34} = - \int_{V_2}^{V_1} P_2 dV = -P_2(V_1 - V_2) = P_2(V_2 - V_1)$$

$$W_{41} = - \int_{V_1}^{V_1} P dV = 0$$

$$W = W_{12} + W_{23} + W_{34} + W_{41} = -P_1(V_2 - V_1) + P_2(V_2 - V_1)$$

$$W = (P_2 - P_1)(V_2 - V_1)$$

$$\Delta E = 0 = Q + W$$

$$\therefore Q = -W = -(P_2 - P_1)(V_2 - V_1) = (P_1 - P_2)(V_2 - V_1)$$

- 1.6
- i) $(T_1, V_1) \rightarrow (T_2, V_1)$ $-\Delta V = 0$
 - ii) $(T_2, V_1) \rightarrow (T_2, V_2)$ $-\Delta T = 0$
 - iii) $(T_2, V_2) \rightarrow (T_1, V_2)$ $-\Delta V = 0$
 - iv) $(T_1, V_2) \rightarrow (T_1, V_1)$ $-\Delta T = 0$

$$W = - \int P dV \quad PV = RT$$

$$P = \frac{RT}{V}$$

$$W = - \int \frac{RT dV}{V} = -RT \ln \left(\frac{V_2}{V_1} \right)$$

$$W_{12} = - \int_{V_1}^{V_1} \frac{RT_1 dV}{V} = 0$$

$$W_{23} = - \int_{V_1}^{V_2} \frac{RT_2 dV}{V} = -RT_2 \ln \left(\frac{V_2}{V_1} \right)$$

$$W_{34} = - \int_{V_2}^{V_2} \frac{RT_2 dV}{V} = 0$$

$$W_{41} = - \int_{V_2}^{V_1} \frac{RT_1 dV}{V} = -RT_1 \ln \left(\frac{V_1}{V_2} \right) = RT_1 \ln \left(\frac{V_2}{V_1} \right)$$

$$W = W_{12} + W_{23} + W_{34} + W_{41} = R(T_1 - T_2) \ln \left(\frac{V_2}{V_1} \right)$$

$$Q = -W = R(T_2 - T_1) \ln \left(\frac{V_2}{V_1} \right)$$

1.7 Calculate the change in internal energy when 1 mole of a liquid water at 1 atm and 100°C is evaporated to water vapour at the same pressure and temperature, given that the molar volumes of the liquid and the vapour under these conditions are $18.8\text{ cm}^3/\text{mol}$ and $3.02 \cdot 10^4\text{ cm}^3/\text{mol}$, and that the latent heat of evaporation is $4.06 \cdot 10^4\text{ J/mol}$.

$$P_1 = 1\text{ atm}$$

$$P_2 = 1\text{ atm}$$

$$T_1 = 100^\circ\text{C} = 373.15\text{ K}$$

$$T_2 = 373.15\text{ K}$$

$$dQ = dU + PdV$$

$$\text{Since } PV = RT \rightarrow V_1 = V_2$$

$$dQ = \left(\frac{4.06 \cdot 10^4\text{ J}}{\text{mol}} \right) (1\text{ mol}) = 4.06 \cdot 10^4\text{ J} = 40600\text{ Nm}$$

$$-PdV = -(1\text{ atm}) (18.8\text{ cm}^3 - 3.02 \cdot 10^4\text{ cm}^3)$$

$$= \left(\frac{101325\text{ N}}{\text{m}^2} \right) (30,181,2\text{ cm}^3) \left(\frac{1\text{ m}}{100\text{ cm}} \right)^3 = 3058\text{ Nm}$$

$$\Delta E = Q + W = dQ + PdV = 40600\text{ Nm} - 3058\text{ Nm}$$

$$= 37542\text{ J}$$

$$= 3.754 \cdot 10^4\text{ J (for 1 mole)}$$

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