Lecture 16 Blackbody Radiation 4 November 2008

Cosmic microwave background radiation is an example of blackbody radiation

Electromagnetic radiation is in thermal equilibrium with matter. In a cavity it is in equilibrium with the walls of the cavity. The cavity is a vaccuum. The emitted radiation by the atoms is photons.

Bremstralang Radiation is from e^- accelerating.

The cavity has an infinite number of points - degrees of freedom - normal modes.

$$\varepsilon_\alpha = \frac{\hbar\omega_\alpha}{e^{\hbar\omega_\alpha/kT}-1} + \underbrace{\frac{\hbar\omega_\alpha}{2}}_{\text{ignore}} = \text{energy of one mode}$$

$$E = \sum_{\alpha=1}^{\infty} \varepsilon_\alpha = \sum_{\alpha=1}^{\infty} \frac{\hbar\omega_\alpha}{e^{\hbar\omega_\alpha/kT}-1} = \int_0^\infty \underbrace{\left(\frac{\hbar\omega}{e^{\hbar\omega/kT}-1}\right)}_{\text{one normal modes}} \underbrace{f(\omega)d\omega}_{\text{number of oscillators whose frequency between } \omega \text{ to } \omega+d\omega}$$

$$f(\omega)d\omega = \begin{cases} \dfrac{V\omega^2 d\omega}{2\pi^2}\left[\dfrac{1}{V_L^3} + \dfrac{2}{V_T^3}\right] & \text{elastic waves} \\[2ex] \dfrac{V\omega^2 d\omega}{\pi^2 V_T^3} & V_L = 0 \text{ debye's model} \\[2ex] \dfrac{V\omega^2 d\omega}{\pi^2 c^3} & V_T \to c \text{ electromagnetic radition} \end{cases}$$

V = volume of the cavity

$$E = \frac{V\hbar}{\pi^2 c^3}\int_0^\infty \frac{\omega^3 d\omega}{(e^{\hbar\omega/kT}-1)}$$

With no quantization, classical limit $\hbar \to 0$, $x = \frac{\hbar\omega}{kT} << 1$

$$\lim_{\hbar \to 0} \frac{\hbar\omega}{e^{\hbar\omega/kT}-1} = \frac{\hbar\omega}{1+\frac{\hbar\omega}{kT}-1} = kT$$

According to the equipartition theorem, every quadratic term in the Hamiltonian contributes $\frac{1}{2}kT$

$$E = \frac{V\hbar kT}{\pi^2 c^3}\int_0^\infty \omega^2 d\omega \to \text{diverges and becomes infinite}$$

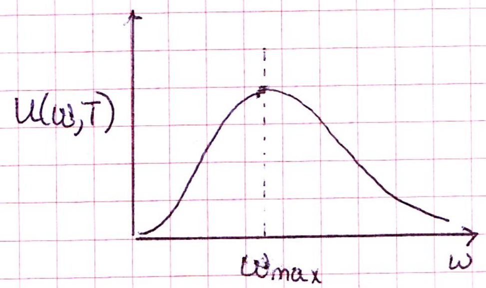
Plank Solved the paradox of the equipartition theorem, that is why $\hbar$ doesn't go to zero.

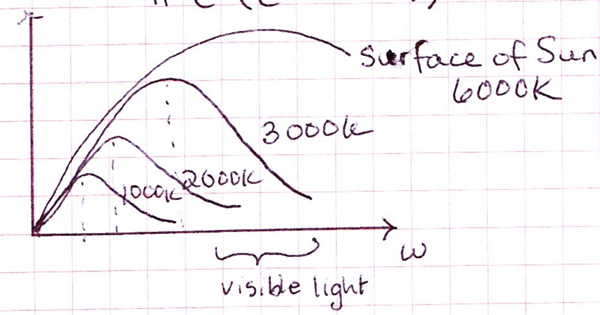
Blackbody Radiation

$$E(\omega,T)\,d\omega = \frac{V\hbar\omega^3 d\omega}{\pi^2 c^3(e^{\hbar\omega/kT}-1)}$$ spectral distribution of energy in the cavity

$$U(\omega,T)d\omega = \frac{E(\omega,T)\,d\omega}{V} = \text{spectral energy density} = \frac{\hbar\omega^3 d\omega}{\pi^2 c^3(e^{\hbar\omega/kT}-1)}$$

Blackbody Function $\quad U(\omega,T) = \dfrac{\hbar\omega^3}{\pi^2 c^3(e^{\hbar\omega/kT}-1)}$





at small ω, $U \propto \omega^3$

$\omega \gg 1$, $U(\omega,T) \to 0$

ω_{max} is the frequency where spectral energy is a maximum

$$U(\omega,T) = \frac{\hbar\omega^3}{\pi^2 c^3(e^{\hbar\omega/kT}-1)}$$

$\dfrac{\partial U}{\partial \omega} = 0$ to get ω_{max}

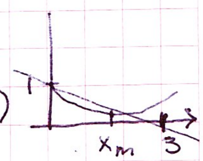
$$\frac{\hbar}{\pi^2 c^3}\left[\frac{3\omega^2}{(e^{\hbar\omega/kT}-1)} + \frac{-\omega^3 \frac{\hbar}{kT} e^{\hbar\omega/kT}}{(e^{\hbar\omega/kT}-1)^2}\right] = 0$$

$$\rightarrow \left(\frac{\hbar\omega}{kT} - 3\right)e^{\hbar\omega/kT} = 3$$

$$3 - \frac{xe^x}{e^x-1} = 0$$

$$1 - \frac{x}{3} = e^{-x}$$

$$f_1(x) = f_2(x)$$



draw a graph, $x_m = 2.822$ or $x_m = 0$

$$\omega_{max} = \frac{x_{max}kT}{\hbar} = \frac{2.822\,kT}{\hbar}$$ Wein's displacement law $\omega_{max} \propto T$ (first determination of Planck's constant)

or $\lambda_{max}T = \text{constant}$

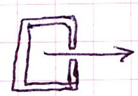
Total energy density in cavity

let $x = \hbar\omega/kT$

$$U(T) = \int U(T,\omega)d\omega = \frac{\hbar}{\pi^2 c^3}\int_0^\infty \frac{\omega^3 d\omega}{e^{\hbar\omega/kT}-1} = \frac{\hbar}{\pi^2 c^3}\left(\frac{kT}{\hbar}\right)^4\int_0^\infty \frac{x^3dx}{e^x-1} = \left(\frac{\pi^2 k^4}{\hbar^3 c^3 15}\right)T^4$$

$U(T) \propto T^4$ Stefan-Boltzmann Law

Cavity Radiation - make a small hole and radiation will come out of it.



$$I = \frac{c}{4}U(t) = \sigma_B T^4 \qquad \sigma_B = \frac{\pi^2 k^4}{60(\hbar c)^3} = 5.67\cdot 10^{-8}\, J s^{-1} m^{-2} K^{-4}$$