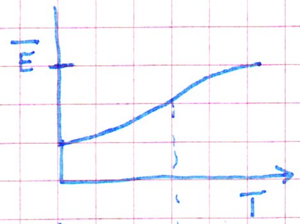
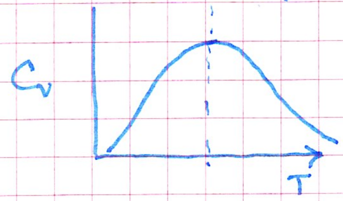
Lecture 11 Going Over Exam I

I.

a)



$$\overline{E} = \frac{E_1+E_2+E_3}{3} \quad @\, T\to\infty \qquad \overline{E} = NE_1 \quad @\, T = 0K$$



$$C_v = \frac{\partial E}{\partial T}$$

b) Calculate exact expressions

Z_1 = partition function for the subsystem $= \sum_r e^{-E_r/kT} = e^{-E_1/kT} + e^{-E_2/kT} + e^{-E_3/kT}$

$$p_r = \frac{1}{Z_1} e^{-E_r/kT}$$

$$\overline{E}_1 = \sum_r p_r E_r = \frac{E_1 e^{-E_1/kT} + E_2 e^{-E_2/kT} + E_3 e^{-E_3/kT}}{Z_1}$$

$$= \frac{\cancel{e^{-E_1/kT}}\left(E_1 + E_2 e^{-(E_2-E_1)/kT} + E_3 e^{-(E_3-E_1)/kT}\right)}{\cancel{e^{-E_1/kT}}\left(1 + e^{-(E_2-E_1)/kT} + e^{-(E_3-E_1)/kT}\right)}$$

$$\overline{E} = N\overline{E}_1 = \frac{N\left(E_1 + e^{-\varepsilon_1/kT} + e^{-\varepsilon_2/kT}\right)}{\left(1 + e^{-\varepsilon_1/kT} + e^{-\varepsilon_2/kT}\right)} \qquad \varepsilon_1 = E_2 - E_1 \quad \varepsilon_2 = E_3 - E_1$$

$$C_v = \frac{\partial \overline{E}}{\partial T} = \frac{N\left(E_1 + e^{-\varepsilon_1/kT} + e^{-\varepsilon_2/kT}\right)(-1)\left[e^{-\varepsilon_1/kT}\left(\frac{-\varepsilon_1}{k}\right)\left(\frac{-1}{T^2}\right) + e^{-\varepsilon_2/kT}\left(\frac{-\varepsilon_2}{k}\right)\left(-\frac{1}{T^2}\right)\right]}{\left(1 + e^{-\varepsilon_1/kT} + e^{-\varepsilon_2/kT}\right)^2}$$

$$+ \frac{N\left[e^{-\varepsilon_1/kT}(-\varepsilon_1/k)(-1/T^2) + e^{-\varepsilon_2/kT}(-\varepsilon_2/k)(-1/T^2)\right]}{\left(1 + e^{-\varepsilon_1/kT} + e^{-\varepsilon_2/kT}\right)}$$

$$= \cancel{\frac{N}{kT^2}\left(1 + e^{-\varepsilon_1/kT} + e^{-\varepsilon_2/kT}\right)\left(\varepsilon_1 e^{-\varepsilon_1/kT} + \varepsilon_2 e^{-\varepsilon_2/kT}\right) - \left(E_1 + E_2 e^{-\varepsilon_1/kT} + E_3 e^{-\varepsilon_2/kT}\right)}$$

$$C_v = \frac{N}{kT^2}\,\frac{\left(\varepsilon_1 e^{-\varepsilon_1/kT} + \varepsilon_2 e^{-\varepsilon_2/kT}\right)\left[(1-E_1) + (1-E_2)e^{-\varepsilon_1/kT} + (1-E_2)e^{-\varepsilon_2/kT}\right]}{\left(1 + e^{-\varepsilon_1/kT} + e^{-\varepsilon_2/kT}\right)^2}$$

easier to just write:

$$\overline{E} = N\frac{\sum_r E_r e^{-E_r/kT}}{\sum_r e^{-E_r/kT}} = \frac{N}{Z}\sum E_r e^{-E_r/kT} \qquad \frac{\partial(1/Z)}{\partial T} = -\frac{1}{Z^2}\left(-E_r/k\right)\left(-1/T^2\right) = \frac{-E_r}{Z^2 kT^2}$$

$$C_v = \frac{\partial \overline{E}}{\partial T} = \cancel{\frac{N}{Z^2}\frac{\partial Z}{\partial T}}\; N\frac{\partial(1/Z)}{\partial T}\sum E_r e^{-E_r/kT} + \frac{N}{Z}\sum E_r e^{-E_r/kT}\left(\frac{-E_r}{k}\right)\left(\frac{-1}{T^2}\right) = \frac{N}{kT^2}\,\frac{\sum_r E_r e^{-E_r/kT}}{\left(\sum_r e^{-E_r/kT}\right)^2}$$

II $\boxed{\begin{matrix} P_1 & T \\ V_1 & N \end{matrix} \Big| \begin{matrix} P_2 & T \\ V_2 & N \end{matrix}}$ $\boxed{P\ V_0\ T\ 2N}$

$P_1V_1 = NkT$
$P_2V_2 = NkT$
$V_1 + V_2 = V_0 = \frac{2NkT}{P} \rightarrow \frac{2}{P} = \frac{1}{P_1} + \frac{1}{P_2} = \frac{P_1+P_2}{P_1P_2} \rightarrow P = \frac{2P_1P_2}{P_1+P_2}$

how muchhas entropy changed?

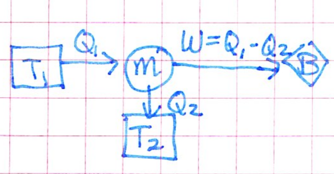
$\Delta S = \Delta S_1 + \Delta S_2 \qquad dE = dQ + dW = 0 \rightarrow dQ = PdV$

$\Delta S_1 = \int \frac{dQ}{T} = \int \frac{PdV}{T} = -\int \frac{VdP}{T} = -Nk\int_{P_1}^{P_f} \frac{dP}{P} = -Nk\ln\left(\frac{P_f}{P_1}\right) \qquad \Delta S_2 = -Nk\ln\left(\frac{P_f}{P_2}\right)$

$$\Delta S = -Nk\ln\left(\frac{P_f^2}{P_1P_2}\right) = -Nk\ln\left[\frac{4(P_1P_2)^2}{P_1P_2(P_1+P_2)^2}\right] = Nk\ln\left[\frac{(P_1+P_2)^2}{4P_1P_2}\right]$$

Since $(P_1+P_2)^2 > 4P_1P_2$ ∴ total entropy change is positive.

III. $T_1 \geq T_2$



lets assume $\Delta S_m = 0 \quad \Delta S_B = 0$ entropy change is in the reservoir

$Q_1 = C(T_1 - T_f) \qquad Q_2 = C(T_f - T_2) \qquad W = Q_1 - Q_2 = C(T_1 + T_2 - 2T_f)$

reversible: $\Delta S_{TOT} \geq 0 \quad \underbrace{C\ln\frac{T_f}{T_1}}_{\text{decreases}} + \underbrace{C\ln\frac{T_f}{T_2}}_{\text{increases}} \geq 0 \rightarrow \ln\frac{T_f^2}{T_1T_2} \geq 0 \qquad T_f \geq \sqrt{T_1T_2}$

as T_f increases, the work available increases

$W \leq C(T_1 + T_2 - 2\sqrt{T_1T_2}) \qquad W_{max} \leq C(\sqrt{T_1} - \sqrt{T_2})^2$

What is the maximum efficiency (because heat reservoir don't have infinite capacities $\quad \eta_{max} = \frac{W_{max}}{Q_1} = \frac{C(\sqrt{T_1} - \sqrt{T_2})^2}{C(T_1 - \sqrt{T_1T_2})} = \left(\frac{\sqrt{T_1} - \sqrt{T_2}}{\sqrt{T_1}}\right)$

$\eta_c = \frac{T_1 - T_2}{T_1} = .64$

IV $\mu = 10^{-26}\ Am^2 \qquad p_+ = \frac{e^x}{e^x + e^{-x}} = .8 \rightarrow 2e^x = 8e^{-x} \rightarrow e^{2x} = 4 \rightarrow x = \frac{1}{2}\ln 4$

$x = \frac{\mu B}{kT} = \frac{1}{2}\ln 4 \rightarrow B = \left(\frac{kT}{\mu}\right)\frac{1}{2}\ln 4 = \frac{1.38\cdot10^{-23}JK^{-1}(10^{-2}K)\ln 4}{2(10^{-26}Am^2)}$

$B = \frac{(1.38)(10)(\ln 4)}{2}$ Tesla