Lecture 10 Review for Exam

Examples of Quasistatic Processes
Adiabatic $\Delta Q = 0$
Isothermal $\Delta T = 0$

1st Law of Thermodynamics: $dE = đQ + đW$
Heat is a transfer of energy
internal energy is the motion of atoms/molecules when Center of Mass is stationary

2nd Law of Thermodynamics: $\Delta S_{TOTAL} \geq 0$ natural processes of isolated systems. At equilibrium, S is a maximum.
When $\Delta S = 0$, the process is reversible. $S = k \ln \Omega$
means that energy is constant in equilibrium
$k \rightarrow$ 'Energy/Temperature' Ω = number of microstates
it is a statistical weight
S is an additive property. The general definition is

$$S = -k \sum_r p_r \ln p_r$$

p_r is the probability of a particular microstate, the partition function is useful for determining p_r

$$p_r = \frac{e^{-E_r/kT}}{Z} = \frac{e^{-\beta E_r}}{Z} \qquad Z = \sum_r e^{-E_r/kT} = \text{partition function}$$

Paramagnetic System is an example:
system: magnetic moment of a single atom
heat bath: solid 10^{23} atoms
fundamental theorem (all microstates are equally probable) $E = \pm \vec{\mu} \cdot \vec{B}$

$$Z_1 = e^{+\mu B/kT} + e^{-\mu B/kT}$$

spin ½ system $\{-\mu, +\mu\}$
Spin 0 system $\{-\mu, 0, +\mu\}$

$$p_+ = \frac{e^{+\mu B/kT}}{e^{\mu B/kT} + e^{-\mu B/kT}} = \frac{e^x}{e^x + e^{-x}} \qquad p_- = \frac{e^{-x}}{e^x + e^{-x}}$$

normally all moments are randomly oriented. $\vec{\tau} = \vec{\mu} \times \vec{B}$ vs. thermal T
if $p_+ \geq 0.75$ then 75% oriented what is x?

$$\frac{e^x}{e^x + e^{-x}} \geq \frac{3}{4}$$

$$4e^x \geq 3e^x + 3e^{-x}$$

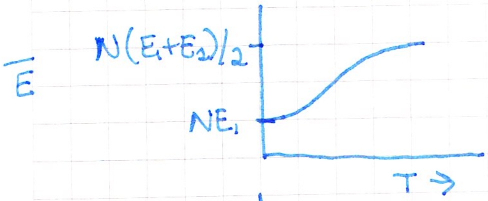
$$e^{2x} \geq 3$$

$$2x \geq \ln 3$$

$$x \geq \tfrac{1}{2} \ln 3 \qquad \frac{\mu B}{kT} \geq \tfrac{1}{2} \ln 3 \text{ if } T = .1K \quad B = \frac{\tfrac{1}{2}\ln 3\, k(.1)}{\mu\,(A/m^2)} = \text{Tesla}$$

NOTE BOOK

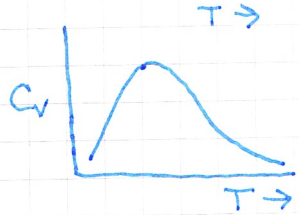
$\overline{E} = N_1E_1 + N_2E_2$ $\quad \lim_{T\to 0} \overline{E} = NE_1$ $\quad \langle E \rangle = \frac{E_1+E_2}{2}$ $\quad \lim_{T\to\infty} \overline{E} = N\left(\frac{E_1+E_2}{2}\right)$



To solve it exactly:

$\overline{E} = \sum_r p_r E_r = p_1E_1 + p_2E_2$

$\overline{E} = N\left(\frac{E_1 e^{-E_1/kT} + E_2 e^{-E_2/kT}}{e^{-E_1/kT} + e^{-E_2/kT}}\right)$



$C_v = \frac{\partial \overline{E}}{\partial T}$

Thermodynamic Potentials: $\quad F = E - TS \qquad G = E + PV - TS$

can show that $P = \left(\frac{\partial F}{\partial V}\right)_T \qquad S = -\left(\frac{\partial F}{\partial T}\right)_V$

$đW = -PdV$ for a reversible quasistatic process

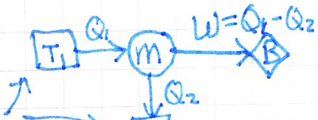
$đQ = TdS \qquad \Delta S = \int^2 \frac{đQ}{T}$ entropy change for a process

isothermal
$PV = NkT \rightarrow$ if T is fixed $\rightarrow PdV = -VdP$

$\Delta S = \int \frac{PdV}{T} = -\int_1^2 \frac{VdP}{T} = -Nk\int_1^2 \frac{dP}{P}$

Heat Engines

Carnot Cycle



* same ~~infinite~~ heat capacity

powerplant

m - no entropy change in cycle
B - no "
T_1 - entropy decrease
T_2 - entropy increase

$\Delta S_{TOT} = -\frac{Q_1}{T_1} + \frac{Q_2}{T_2} \geq 0 \Rightarrow \frac{Q_2}{Q_1} \geq \frac{T_2}{T_1}$

$\eta = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1} \leq 1 - \frac{T_2}{T_1}$

* fintite heat capacity means energy will stop flowing when $T_1 = T_2$

$T_1 \rightarrow T_F \qquad T_2 \rightarrow T_F$

$Q_1^{TOT} = C(T_1 - T_F)$
$Q_2^{TOT} = C(T_F - T_2)$

$W^{TOT} = Q_1^{TOT} - Q_2^{TOT} = C(T_1 + T_2 - 2T_F)$

What can we say about this final temperature?

$\Delta S_1 = \int \frac{đQ}{T} = \int_{T_1}^{T_f} \frac{CdT}{T} = C\ln\left(\frac{T_f}{T_1}\right) \quad \Delta S_2 = C\ln\left(\frac{T_f}{T_2}\right)$

can assume C is a constant of T, but it isn't

$\Delta S = \Delta S_1 + \Delta S_2 = C\left[\ln\frac{T_f}{T_1} + \ln\frac{T_f}{T_2}\right] \geq 0$

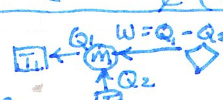
for quasistatic $\quad T_f \geq \sqrt{T_1T_2}$

look at the expression for total work
the maximum work that can be done

$W^{TOT} \leq C(T_1 + T_2 - 2\sqrt{T_1T_2})$
$W^{TOT} \leq C(\sqrt{T_1} - \sqrt{T_2})^2$

$\eta = \frac{W^{TOT}}{Q_1^{TOT}} \qquad \eta^{max} = \frac{C(\sqrt{T_1} - \sqrt{T_2})^2}{C(T_1 - \sqrt{T_1T_2})}$ efficiency

Refrigerator



$\Delta S_{TOT} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2} \geq 0$

performance: $\eta_{HP} = \frac{Q_1}{W} \qquad \frac{1}{\eta_{HP}} = 1 - \frac{Q_2}{Q_1} \geq 1 - \frac{T_2}{T_1}$

$\eta_{HP} \leq \frac{T_1}{T_1 - T_2}$

3rd Law of Thermodynamics: $\lim_{T\to 0} S = 0$ usually say $\lim_{T\to 0} S = \text{constant}$