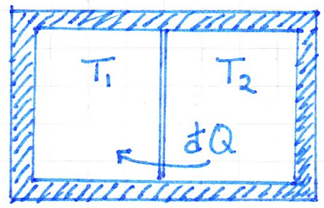
Lecture 8 Heat Engines

(A) Equivalence of the different formulations of the 2^{ND} Law

(1) Heat can't spontaneously flow from a colder body to a hotter body. Need a machine to cause that heat to flow.

(2) For an isolated system (fixed energy E) for any natural process, entropy must increase - natural processes are irreversible.

$T_1 < T_2$



$$\Delta S_{TOTAL} = \frac{đQ}{T_1} - \frac{đQ}{T_2} \geq 0$$

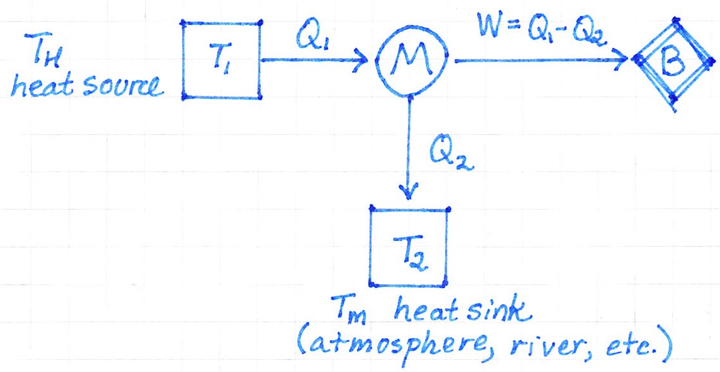
$\Delta S_{TOTAL} < 0$ is impossible!

(3) A process whose only effect is the complete conversion of heat into work can't happen.

(B) Efficiency of Engine

What fraction of heat can be converted to work?

Carnot Engine



work is done on body B

Mechanism M works in cycle and returns to initial state
entropy of M doesn't change
entropy of B doesn't change

The entropy change of this engine is entirely in the heat source and the heat sink.

$$\Delta S_{TOTAL} = -\frac{Q_1}{T_1} + \frac{Q_2}{T_2} \geq 0$$

$$\therefore \frac{Q_2}{T_2} \geq \frac{Q_1}{T_1} \quad \text{and} \quad \frac{Q_2}{Q_1} \geq \frac{T_2}{T_1}$$

Definition of Efficiency:

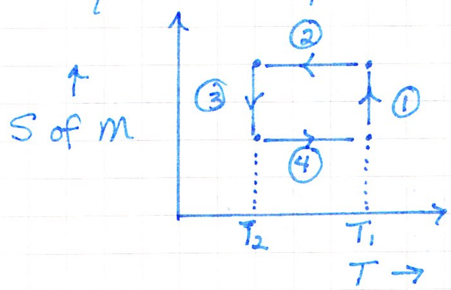
$$\eta = \frac{W}{Q} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \leq 1 - \frac{T_2}{T_1}$$

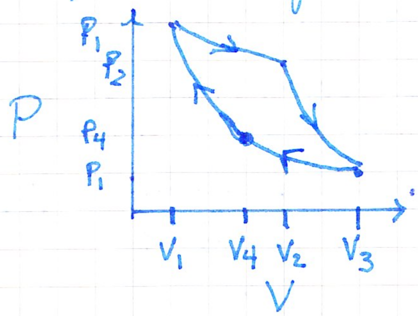
equality only if the process is reversible

$$\eta_{max} = \frac{T_1 - T_2}{T_1}$$

A Carnot cycle is a reversible heat engine.

A quasistatic process is always represented by a closed curve.





$\eta = \dfrac{Q_1 - Q_2}{Q}$

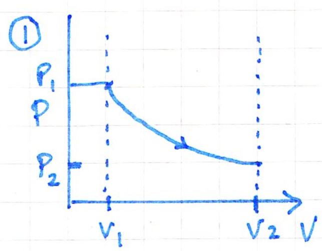
$\eta = \dfrac{nR\Theta_1 \ln\left(\frac{V_2}{V_1}\right) - nR\Theta_2 \ln\left(\frac{V_3}{V_4}\right)}{nR\Theta_1 \ln(V_2/V_1)}$

if $\ln(V_2/V_1) = \ln(V_3/V_4)$

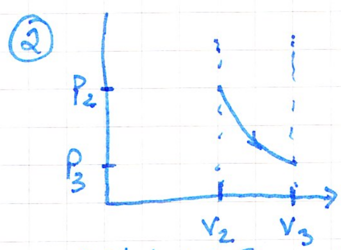
then $\eta = \dfrac{\Theta_1 - \Theta_2}{\Theta_1}$

below show that $V_2/V_1 = V_3/V_4$

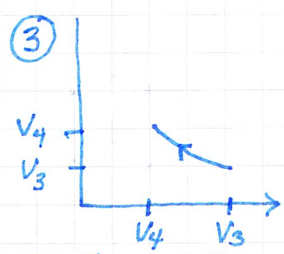
Process	Energy Flow	Entropy Change	Type of Process
①	$Q_1 \rightarrow M$	$S_m \uparrow$	Isothermal Expansion $W = -\int P\,dV = -nR\Theta \int_1^2 \frac{dV}{V}$ Work ON gas is negative
②	Work done, T decreases		Adiabatic Expansion
③	$Q_2 \leftarrow M$		Isothermal Compression
④	heated		Adiabatic Compression $\left.\begin{matrix} PV^\gamma = \text{constant} \\ PV = nRT \end{matrix}\right\}$ $\Theta V^{\gamma-1} = \text{constant}$



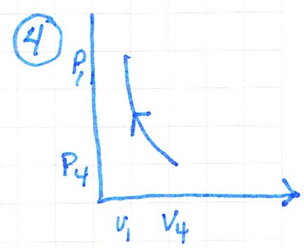
isothermal expansion



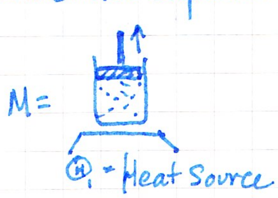
Adiabatic Expansion



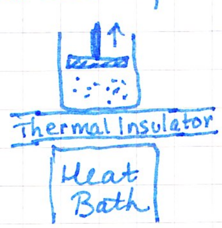
Isothermal Compression



M =



M =



$\Delta E = \Delta Q + \Delta W = 0$

$\Delta Q = -\Delta W = -W_1 = Q_1$

$W_1 = -\int P\,dV = nR\Theta_1 \ln\left(\frac{V_2}{V_1}\right)$

$PV^\gamma = \text{constant}$

$\Theta_1 V_2^{\gamma-1} = \Theta_2 V_3^{\gamma-1}$

$\therefore W_2 = -nR\Theta_2 \ln\left(\frac{V_4}{V_3}\right)$

$\therefore Q_2 = \Delta W = -nR\Theta_2 \ln\left(\frac{V_4}{V_3}\right)$

$V_4 < V_3$

$\therefore W_2 > 0$

$\Delta E = \Delta Q + \Delta W = 0$

$\Delta Q = -\Delta W$

heat supplied to gas is negative

$\therefore$ heat is given to sink

$\Theta_2 V_4^{\gamma-1} = \Theta_1 V_1^{\gamma-1}$

$\therefore \dfrac{V_2}{V_1} = \dfrac{V_3}{V_4}$