Today we will continue studying the 2-Particle system:

$$\hat{H} = \underbrace{\frac{\hat{\mathcal{P}}^2}{2M}}_{\text{Center of Mass}} + \underbrace{\frac{\hat{p}_r^2}{2\mu} + \underbrace{\frac{\hat{L}^2}{2\mu r^2} - \frac{Z'e^2}{r}}_{\text{Effective Potential}}}_{\text{Relative Hamiltonian for a hydrogenic atom of atomic number Z}}$$

Here $-\frac{Z'e^2}{r}$ is the Coloumb potential

The effective potential $Z' = \frac{Z}{\varepsilon}$ and ε is permittivity.

The eigenenergies are:

$$E_{k,n,l,m} = \frac{\hbar k^2}{2M} - \frac{Z^2\Re}{n^2}$$

$\Re =$ Rydberg constant $= \frac{\mu e^4}{2\hbar^2}$

For a Hydrogen atom, $\Re = 13.6$ eV

The degeneracy of E_n of l:

$$D(n) = \sum_{l=0}^{n-1}(2l+1) = n^2 \quad \text{where } n = 1,2,3,\cdots$$

We can write the eigenfunctions this way:

$$\psi_{k,n,l,m} = \tfrac{1}{\sqrt{V}} e^{-i\vec{k}\cdot\vec{R}} R_{n,l}(r) Y_l^m(\theta,\phi)$$

Where V is the volume of the space.

Table 10.4 is explained in class

Principle Quantum Number *n*	0	1		2		
Angular Component *l* *l=0,1,2,…,n-1*	0	0	1	0	1	2
Spectroscopic Notation of State	1S	2S	2P	3S	3P	3D
$m_l = -l,-l+1,\ldots,0,\ldots,l-1,l$	0	0	-1,0,1	0	-1,0,1	-2,-1,0,1,2
Degeneracy of State n^2	1	4		9		
Degeneracy considering spin	2	8		18		

Consider Na, with Z=11. The shells are filled in order $1s^2\ 2s^2\ 2p^6\ 3s^1$. This and other atoms in the 1A series can be treated as a 2 particle model – one particle is the $1s^2\ 2s^2\ 2p^6$ core and the other is the $3s^1$ electron.

Let's look at the lowest energy level. *n=1, l=0, m=0*

$$R_{10} = \frac{A_{10}u_{10}}{r}$$

$$u_{10} = A_{10}e^{-\rho/2}\rho$$

$$\rho \equiv 2\frac{Z}{a_0 n} \equiv \frac{2r}{a_0}$$

$$\int |R_{10}|^2 r^2 dr = 1$$

$$\frac{a_0|A_{10}|^2}{2}\int_0^\infty e^{-\rho}\rho^2 d\rho = 1$$

$$|A_{10}| = \frac{1}{a_0}$$

$$R_{10} = \frac{A_{10}u_{10}}{r} = \frac{2}{(a_0)^{3/2}}e^{-r/a_0}$$

Ground state of the atom can move together:

$$\psi_{l,1,0,0} = \frac{1}{\sqrt{V}}e^{-i\vec{k}\cdot\vec{R}}\frac{2}{(a_0)^{3/2}}e^{-r/a_0}\frac{1}{\sqrt{4\pi}}$$

$\varphi_g = Ae^{-r/a_0}$ ground state wave function of electron

Next we discussed Table 10.5

Spectroscopic Notation	Normalized Time-Independent Eigenstates
1S	$\varphi_{100} = \dfrac{2}{(a_0)^{3/2}} e^{-r/a_0} Y_0^0(\theta,\phi)$
2S	$\varphi_{200} = \dfrac{2}{(2a_0)^{3/2}} (1 - r/2a_0) e^{-r/2a_0} Y_0^0(\theta,\phi)$
2P	$\begin{pmatrix} \varphi_{211} \\ \varphi_{210} \\ \varphi_{21-1} \end{pmatrix} = \dfrac{1}{\sqrt{3}(2a_0)^{3/2}} \dfrac{r}{a_0} e^{-r/2a_0} \begin{pmatrix} Y_1^1(\theta,\phi) \\ Y_1^0(\theta,\phi) \\ Y_1^{-1}(\theta,\phi) \end{pmatrix}$
3S	$\varphi_{300} = \dfrac{2}{3(2a_0)^{3/2}} \left(3 - 2r/a_0 + 2(r/3a_0)^2\right) e^{-r/3a_0} Y_0^0(\theta,\phi)$
3P	$\begin{pmatrix} \varphi_{311} \\ \varphi_{310} \\ \varphi_{31-1} \end{pmatrix} = \dfrac{4\sqrt{2}}{9(3a_0)^{3/2}} \dfrac{r}{a_0} (1 - r/6a_0) e^{-r/3a_0} \begin{pmatrix} Y_1^1(\theta,\phi) \\ Y_1^0(\theta,\phi) \\ Y_1^{-1}(\theta,\phi) \end{pmatrix}$
3D	$\begin{pmatrix} \varphi_{322} \\ \varphi_{321} \\ \varphi_{320} \\ \varphi_{32-1} \\ \varphi_{32-2} \end{pmatrix} = \dfrac{2\sqrt{2}}{27\sqrt{5}(2a_0)^{3/2}} \left(\dfrac{r}{a_0}\right)^2 e^{-r/3a_0} \begin{pmatrix} Y_2^2(\theta,\phi) \\ Y_2^1(\theta,\phi) \\ Y_2^0(\theta,\phi) \\ Y_2^{-1}(\theta,\phi) \\ Y_2^{-2}(\theta,\phi) \end{pmatrix}$

Problem to solve. What is the ground state energy for each of the following 2-Particle systems?

1.) H_2, a deuteron and an electron
2.) He^+, a single ionized Helium atom
3.) Positronium, a bound positron and electron
4.) Exciton, with ε=10 (dielectric constant)

$$E_{0,1,0,0} = \cancel{\frac{\hbar^2 k^2}{2M}}^{\text{ignore CM system}} - \frac{Z'^2 \Re}{n^2}$$

$$\Re = \left(\frac{1}{4\pi\varepsilon}\right)^2 \frac{\mu e^4}{2\hbar^2} = \frac{\left(8.99\cdot 10^9 \text{ N m}^2\text{ C}^{-2}\right)^2 \left(9.109\cdot 10^{-31}\text{kg}\right)\left(1.6022\cdot 10^{-19}\text{C}\right)^4}{2\left(1.0546\cdot 10^{-34}\text{N m s}\right)^2 \left(1.6022\cdot 10^{-19}\text{ J eV}^{-1}\right)} = 13.6 \text{ eV}$$

$m_e=9.109\cdot 10^{-31}$ kg
$m_p=1.6726\cdot 10^{-27}$ kg
$m_n=1.675\cdot 10^{-27}$ kg

	$Z' = \frac{Z}{\varepsilon}$	m1	m2	$\mu = \frac{m1\cdot m2}{m1+m2}$ kg	$\Re = \left(\frac{1}{4\pi\varepsilon}\right)^2 \frac{\mu e^4}{2\hbar^2}$ eV	$E = \frac{-Z'^2\Re}{n^2}$ eV
1	1	m_n+m_p	m_e	9.10652e-031	13.608698	-13.608698
2	2	$2m_p$	m_e	9.10652e-031	13.608698	-54.434782
3	1	m_e	m_e	4.5545e-03	6.806201	-6.806201
4	$\frac{1}{10}$	m_e	m_e	4.5545e-03	6.806201	-0.068062