26 Feb 2009 Polarization, Linear + Elliptic

Residual Absorption  Beer's Law $e^{-\alpha t} I_0$ ← thickness

Malus' Law $\frac{I}{I_m} = \cos^2\theta$  when there is no residual absorption

Beer's Law + Malus' Law $\frac{I}{I_m} = e^{-\alpha t}\cos^2\theta$

Can use birefringent materials anisotropy to (1) induce phase shift $\pm\pi/2$
or (2) left or right circularly polarized light - elliptically

"wave plate" give relative phase shifts between orthogonal components.

$\Delta\phi = 180^\circ \Rightarrow$ flip of $\vec{E}$ polarization $90^\circ$ flip of polarization

Phase Retarder - dial it in - two wedges - effectively change the thickness of the wedges - arbitrary phase shift
= "Phase Compensators"

Another consequence of Polarized Light:

Angular Momentum comes from $\vec{E}$-field

Linear Polarization → no net angular momentum

Power $\propto \tau\omega \propto \left(\frac{dL}{dt}\right)\omega$  $\boxed{E = \hbar\omega}$

Power = $\frac{dE}{dt}$  $L = \pm n\hbar$  linearly polarized light → no angular momentum
circularly polarized light does carry angular momentum.

Selection Rules $\Delta L = \pm 1$  $\Delta m = 0, \pm 1$  Dipole Transitions Atomic Systems
↑ circularly polarized light

Prepare a certain quantum state
Depopulate  Optical Pumping

Equation 2.37 p33  p34

if $E_{oy} = 0$  $\vec{E} = \begin{bmatrix} E_0 e^{i\phi_x} \\ 0 \end{bmatrix}$ in the normalized form $= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

polarized at 45° equal + in phase  $\vec{E} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \Leftrightarrow \begin{bmatrix} E_0 & e^{i\phi} \\ E_0 & e^{i\phi} \end{bmatrix} = E_0 e^{i\phi}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$  polarization at 45°

↑ amplitude  ↑ phase d...

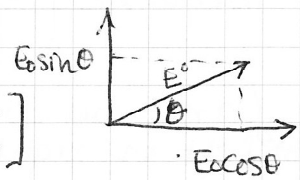
NOTE BOOK

Circularly Polarized Light $E = E_0 (\hat{i} + \hat{j}\, i) e^{i\phi}$

$\sqrt{-1}$ = relative phase shift of $\pi/2$

$$LCP = \begin{bmatrix} 1 \\ +i \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ +i \end{bmatrix}$$

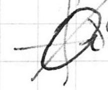
$$1/\sqrt{2} \begin{bmatrix} 1 & +i \end{bmatrix} 1/\sqrt{2} \begin{bmatrix} 1 \\ +i \end{bmatrix} = 1$$




linearly polarized light at arbitrary angle $\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$

two ⊥ components of electric field. the net vector equal amplitudes = 45°

if there is a phase shift (amplitude equal) circularly polarized 

if phase shift + amplitude different elliptically polarized 

eigenvectors or bases vectors of polarization states

equal amounts of RCP and LCP make linear polarized light $\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix} + \begin{bmatrix} 1 \\ -i \end{bmatrix}$

$\begin{bmatrix} 0 \\ i \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix} - \begin{bmatrix} 1 \\ -i \end{bmatrix} = \begin{bmatrix} 0 \\ 2i \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \ldots i$

circular dichroism - absorbs preferentially the circular polarization

$$\begin{bmatrix} 1 \\ i \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + i \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ -i \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} - i \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Isotropic Phase Retarder - Piece of Glass

$$\begin{bmatrix} e^{i\phi} & 0 \\ 0 & e^{i\phi} \end{bmatrix} \qquad \underbrace{\begin{bmatrix} e^{-\alpha d} & 0 \\ 0 & e^{-\alpha d} \end{bmatrix}}_{\text{absorption}}$$

Train of Optical Components: Gives output State of Polarization

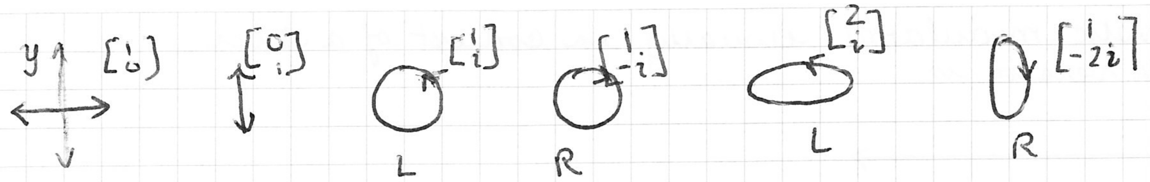
projects out the x-component Chapter 2 accumulates phase...

Reflection, Refraction, etc.

E =

$$\phi = \frac{2\pi}{\lambda} n d$$

common phase factor. index of refraction distance






$i$ = phase shift of 90°

$A^* B = 0$

Jones matrix for ~~H~~ LC pol light $\begin{bmatrix}1 & 0\\0 & i\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}$

Eigenvalues of Jones Matrices

$$\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}A\\B\end{bmatrix} = \lambda\begin{bmatrix}A\\B\end{bmatrix}$$

eigenstates are the states that are unchanged by that element

$$\begin{bmatrix}a-\lambda & b\\c & d-\lambda\end{bmatrix}\begin{bmatrix}A\\B\end{bmatrix} = 0$$

non-trivial Solution $\begin{vmatrix}a-\lambda & b\\c & d-\lambda\end{vmatrix} = 0$

$$(a-\lambda)(d-\lambda) - bc = 0$$

$(a-\lambda)A + bB = 0$ $\quad cA + (d-\lambda)B = 0$ $\quad$ can get $A/B$ corresponding to eigenvalue of $\lambda$

$\begin{vmatrix}1-\lambda & 0\\0 & i-\lambda\end{vmatrix}\begin{bmatrix}A\\B\end{bmatrix} = 0 \quad (1-\lambda)(i-\lambda) = 0 \quad \lambda = 1, i$

$(i-1)B = 0$
$B = iB \Rightarrow B = 0$
$A$ = anything

$A\begin{bmatrix}1\\0\end{bmatrix}$ light ~~in~~ polarized x-direction

$(1-i)A + 0B = 0$
$A = iA \Rightarrow A = 0$
$B$ = anything

$B\begin{bmatrix}0\\1\end{bmatrix}$ light polarized in y-direction

$$\begin{vmatrix}1-\lambda & 1\\1 & 1-\lambda\end{vmatrix}\begin{bmatrix} \ \\ \ \end{bmatrix}$$

$$(1-\lambda)^2 - 1 = 0$$

$$1 - 2\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 2\lambda = 0$$

$$\lambda(\lambda - 2) = 0$$

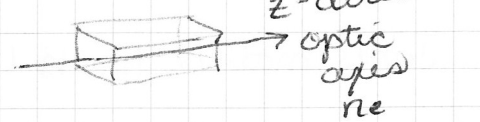
$$\lambda = 0, 2$$

Electrooptic modulator usually in context of a fiber
A bulk version is

Potassium dihydrate Phosphate (KDD)

$n_x = n_y = n_o$ Uniaxial crystal

$n_z = n_e$




$$\begin{bmatrix} n_o & & \\ & n_o & \\ & & n_e \end{bmatrix}$$

$I = I_o \cos^2\theta$

KDD crystal - apply $\vec{E}$-field $\Rightarrow$ very slightly anisotropic

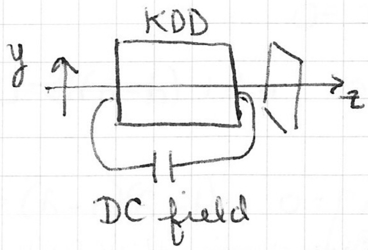
Shows birefringence at 45° between x and y

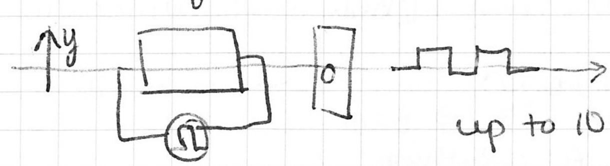
$$\begin{bmatrix} n_x' & 0 & 0 \\ 0 & n_y' & 0 \\ 0 & 0 & n_e \end{bmatrix}$$

$\Delta n = \frac{1}{2} n_o^3 E r$

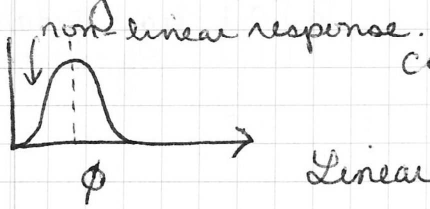
material parameter $\chi^{(2)}$

Pocell's coefficient




analog modulation i want to go home.




can bias
~~to~~ quarter wave plate w $\pi/2$ shift

Linear Electrooptic effect Pockel's effect

make a wave guide
few hundred microns

$$\Delta\phi = \frac{2\pi}{\lambda}\left(\frac{1}{2}n_o^3\right) r\, E_{applied}\, d$$

fixed fixed thickness of crystal

for $\Delta\phi = \pi = \frac{2\pi}{\lambda}\left(\frac{1}{2}n_o^3 r\, E_{app}\right) d = \frac{2\pi\, \Delta n\, d}{\lambda}$ ($\Delta n$)

$E_{app} = \frac{\lambda}{r n_o^3}$

Quadratic E.O. Effect $\Delta n = \lambda^2 K E^2$

p. 192 Kerr $n_{\parallel} - n_{\perp} = K E^2 \lambda_0$

use liquids - apply field to orient

~~fermi's law~~ response time 10ms
field dependent

cooperative effect aligning lower energy # have to align.

distortion of $e^-$ clouds frequency of light   ions lattice
response time is lower
20-30 GHz

Kerr effect : path length
field is transverse to

refractive index changes dipole moment change is small

Nitrobenzene  ⇑ nitrotoluene
a few deByes   $SI = C, m$

cgs Gaussian = e cm
charge in esu's
$e = 4.8 \cdot 10^{-10}$ esu

1 deBye 1e separated by 1 Å   Debye

$d = 4.8 \cdot 10^{-10} \cdot 10^{-8} = 4.8$ deBye   $1 \text{ deBye} = 10^{-8}$

thermal energy is trying to dissociate it

Birefringence phase shift of $\pi$   q-switch [illegible]
feel the love ♡
♡ ♡

Pockel's Cell switch to 0   waveguides

Lithium niobate   Photoresist microlithography

Cutoff dimension = critical dimension optical fiber

linear Birefringence   Circular birefringence
LH vs. RH Circularly polarized light
L-quartz
D-quartz
} two handedness

Quartz uniaxial crystal $n_{\otimes}$ $n_o$

$$\begin{bmatrix} n_o & & \\ & n_o & \\ & & n_e \end{bmatrix}$$

along optic axis - no birefrng. - all dir see $n_o$

in quartz linearly polarized - depends on type of quartz

Optical rotation -
rotates the plane of linearly polarized light

Solution of sugar - also rotates plane of polarization

the reason for circular birefringence

p. 186 "Optical activity" of quartz
p. 187
} Read

Exam 1 week from today, cover up to this class
open book + open notes.

linear bifrigence
Refractive index - [ $\sqrt{\epsilon}$
or quadradic/Kerr effect

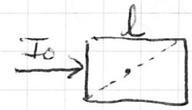
optical activity - circular birefringence (orig molecules have handedness
influences induced dipole
RH or LH DNA isomers
absorption of RCP dif. from LCP

Real + imaginary components of refractive index

quartz is a naturally occuring optically active crystal
along optic axis - no birefringence
plane of polarization
quartz has a small circular birefringence

p. 186 - p 187 $k_r = n_r \frac{\omega}{c}$ $\quad n_L = n_L \frac{\omega}{c}$

magnitude of wave vector




eq. 6.119

after traversing a distance $l$

$$I_o = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \underbrace{\frac{1}{2}\begin{bmatrix} 1 \\ -i \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 \\ i \end{bmatrix}}_{\text{linear combination}} \Rightarrow \frac{1}{2}\begin{bmatrix} i \\ -1 \end{bmatrix} e^{ik_R l} + \frac{1}{2}\begin{bmatrix} 1 \\ i \end{bmatrix} e^{ik_L l} = e^{i(k_R + k_L)l/2}\{\begin{bmatrix} 1 \\ -i \end{bmatrix} e$$

input
x-polarized light beam

eq. 6.120 $\theta = \frac{1}{2}(k_R - k_L)\ell$ $\qquad \psi = \frac{1}{2}(k_R + k_L)\ell$

6.117

6.118

"Specific Rotary Power" $\delta$ for a material

$\delta = (n_R - n_L)\frac{\pi}{\lambda}$ once you determine wavelength, this is the

$\delta(650nm) = 17°/mm$ ; more to absorption edge $\delta(400nm) = 43°/mm$

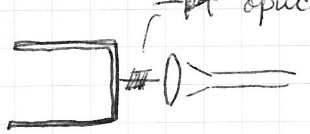
$$\chi = \begin{bmatrix} \chi_{11} & & \\ & \chi_{22} & \\ & & \chi_{33} \end{bmatrix}$$ effective polarizability

$$n = \begin{bmatrix} n_{11} & & \\ & n_{22} & \\ & & n_{33} \end{bmatrix} \Rightarrow \begin{bmatrix} n_o & & \\ & n_o & \\ & & n_e \end{bmatrix} \qquad n = \sqrt{1+\chi}$$

uniaxial material

$\vec{P} = \chi \vec{E}$ $\qquad \chi = \begin{bmatrix} \chi_{11} & i\chi_{12} \\ -i\chi_{12} & \end{bmatrix}$ circularly polarized components when the off-diag. susceptibility are non-zero & imaginary

Optical device remedy: "⊳|" optical diode material allows light to propagate in only one direction



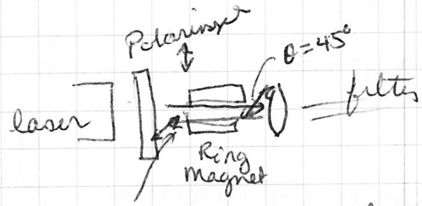
problem: Some light reflects back into laser external feedback makes the laser non-stable $\Rightarrow$ crosstalk errors in transmission

laser Comm fiber optic

Doped Glass Rods $\theta = VB\ell$ 6.134

materials parameter

Verdet's constant




θ=90° doesn't go back into laser

Similarly... V is is proportional to $\dot{r}$ $\propto \frac{dr}{dt}$

we used forced damped harmonic oscillator Lorentz model $m\frac{d^2r}{dt^2} - \gamma\frac{dr}{dt} + \omega_0^2 r = eE$ forcing term

$P = Ne\vec{r}$

$q\vec{v}\times\vec{B}$ look for harmonic solution

Faraday Rotation propagation of light $q\vec{v}\times\vec{B}$

reversal in sign ~~of dir~~ of field

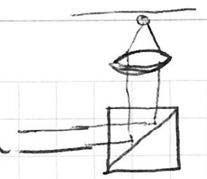
faraday rotator applications

optical isolator here to prevent standing waves removes one direction to get unidirectional ring laser

Ring Laser

used in optical communications

laser



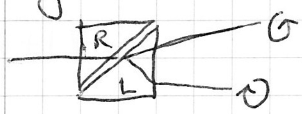
focus on CD to read bits

CD looks for changes in reflectivity

all fiber

wavelength division multiplexing

optically activite material used to sort polarization



dispersion due to $n_R \neq n_L$

Exam Next Class:

Doppler Effect (transverse)

given an E/M harmonic wave - what is the intensity re $n_r$

how much pressure does it apply

force

$\vec{E}$-field due to it

Refractive index Susceptibility

n in general is complex

$\hat{n} = \sqrt{\hat{\varepsilon}}$ $\quad$ n real (no loss) $n = \sqrt{\varepsilon} = \sqrt{1+\chi}$

Ch. 1, 2, 6

Polarization

$\vec{P} = \varepsilon_0 \chi \vec{E}$ isotropic medium

$\vec{P} = \varepsilon_0 \chi \vec{E}$ Scalar $\vec{P} \parallel$ to $\vec{E}$

if anisotropic then $\vec{P} = \varepsilon_0 \chi : \vec{E} = \varepsilon_0 \begin{bmatrix} \chi_{11} & & \\ & \chi_{22} & \\ & & \chi_{33} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$

$\vec{P} = \varepsilon$

Polarization in dif directions

$P_x = \varepsilon_0 \chi_{11} E_x$

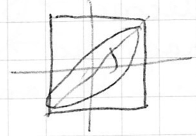
$$\begin{bmatrix} A \\ Be^{i\Delta} \end{bmatrix}$$ one component

$E_x = A\cos(kx - \omega t)$

$E_y = B\cos(kx - \omega t + \Delta)$

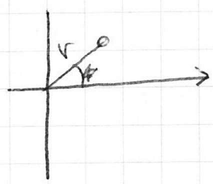
net electric field

like harmonic oscillator problem
the two components have
different amplitudes + dif phase
show path is an ellipse



$y = A\cos(\omega t)$

$x = \cancel{B} \cos(\omega t + \phi)$



$\vec{r} = \hat{i} A\cos\omega t + \hat{j} B\cos(\omega t + \phi)$

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Show the time average value of the Poynting vector

$\langle S \rangle = \langle \mathrm{Re}\, E \times \mathrm{Re}\, H \rangle = \vec{E}_r \times \vec{H}_r$

$\vec{E} = \vec{E}_r + i\vec{E}_i$

$\vec{H} = \vec{H}_r + i\vec{H}_r$

$\vec{E}_r = \frac{1}{2}(\vec{E} + \vec{E}^*)$

$\vec{H}_r = \frac{1}{2}(\vec{H} + \vec{H}^*)$

$\vec{E} = E_r + iE_i$

$\langle S \rangle = \frac{1}{4}\langle (\vec{E} + \vec{E}^*) \times (\vec{H} + \vec{H}^*) \rangle = \frac{1}{4}\left[\vec{E} \times \vec{H}^* + \vec{E}^* \times \vec{H}\right] = \frac{1}{2}\left[\frac{1}{2}\vec{E} \times \vec{H}^* + \frac{1}{2}\vec{E}^* \times \vec{H}\right] = \frac{1}{2}\mathrm{Re}[\vec{E} \times \vec{H}_0]$

time average of $e^{2i\omega t} = \phi$

Office come by for help.

$\vec{P} = \dfrac{\sum p_i}{\sum \Delta v}$ deals with the

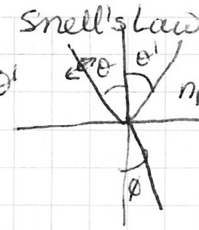
Polarization of light beam is orientation of electric field vector.

light gives rise to polarization of medium

## Reflection & Refraction

Thursday March 11

Find amplitudes of Ref Ref. Trans from Maxwell's Eq's




$\theta = \theta'$

$n_1 \sin\theta = n_2 \sin\phi$

Decompose/Resolve into ⊥ components linear combination the one that is ⊥ to plane of incidence T.E.

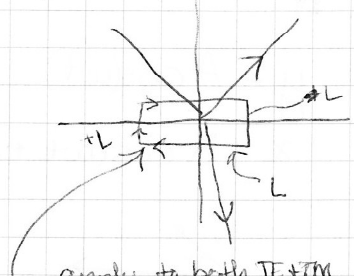
German words - don't make sense...

TE transverse electric → S polarization S+P polarizations

TM transverse magnetic → P polarization

mag field is ⊥ to plane of incidence

Use Maxwell's Equations




have to assume there are no surface currents

HMWK: Algebra for the fresnel coefficients

apply to both TE+TM

Problem 2.19 2.20 2.21

$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial \Phi}{\partial t}$

2.14, 2.15, 2.16, 2.17

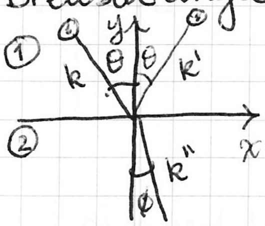
$E_1 L_1 + E L_2 + E$

Analog of potential in optics is refractive index

Tuesday March 24

finish up reflection + transmission coefficients - apply Maxwell's Eq's continuous

Brewster angle




k, k', and k'' lie in the same plane

Snell's Law

$n_1 \sin\theta_1 = n_2 \sin\phi$

Brewster angle $\tan 90° = \infty \therefore r_p \to 0$

$\theta_B + \phi = \pi/2 \Rightarrow \phi = \pi/2 - \theta_B$

$n_1 \sin\theta = n_2 \sin\phi$

$n_1 \sin\theta_B = n_2 \sin(\pi/2 - \theta_B)$

$n_1 \sin\theta_B = n_2 \cos\theta_B$

$\tan\theta_B = \frac{n_2}{n_1} \Rightarrow \theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}(n)$ $n = 1.5 \Rightarrow \theta_B = 54°$

($\theta_c$ comes out to be about 43°)

$\boxed{n = \frac{n_2}{n_1}}$ $n<1$ dense to rare $n>1$ rare to dense

Reflection Coefficients $r_s = \frac{\cos\theta - n\cos\phi}{\cos\theta + n\cos\phi}$ $r_p = \frac{-n\cos\theta + \cos\phi}{n\cos\theta + \cos\phi}$ $\cos\phi = \sqrt{1-\sin^2\phi}$ $= i\sqrt{\frac{\sin^2\phi}{n^2} - 1}$

$E'' = E_0'' e^{i(\vec{k}''\cdot\vec{r} - \omega t)}$ $\vec{k}''\cdot\vec{r} = k_x'' x + k_y'' y$ $\vec{k}\cdot\vec{r} = k'' x \sin\phi + i k'' y \sqrt{\frac{\sin^2\theta}{n^2} - 1}$

$E'' = E_0 e^{-\alpha|y|} e^{i(k'' x \sin\phi)}$ $n = \frac{1}{1.5}$ glass $\alpha = k'' \sqrt{\frac{\sin^2\theta}{n^2} - 1}$

$r_s = \frac{\cos\theta - n^2\sqrt{\sin^2\theta - n^2}}{\cos\theta + n^2\sqrt{\sin^2\theta - n^2}}$ $|r_s|^2 = 1$

if complex conjugate

if $n < 1$

total internal reflection