Tuesday **Optics** Office Hours: Olney 420 x3687 (M 11:30 – 12:30, Th 11:30 – 12:30)
Homework: vector calculus Jayant-Kumar@uml.edu

$\vec{D} = \varepsilon_0 \vec{E}$

In a Vacuum

Gauss' Law in vacuum $\vec{\nabla} \cdot \vec{E} = \cancel{\rho_{tot}}/\varepsilon_0 \Rightarrow 0$ Faraday's Law $\vec{\nabla} \times \vec{E} = -\partial B/\partial t$

No magnetic monopoles $\vec{\nabla} \cdot B = 0$ Generalized Ampere's Law $\vec{\nabla} \times \vec{B} = \mu_0 (\vec{J} + \frac{\partial \vec{D}}{\partial t})$

μ_0 = permeability of vacuum ε_0 = permittivity of vacuum

$\vec{D} = \varepsilon_0 \vec{E}$

take curl of curl: $\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \vec{\nabla}^2 \vec{E}$

faradays law: $\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times (-\partial B/\partial t) = -\partial/\partial t (\vec{\nabla} \times \vec{B}) = -\partial/\partial t (\mu_0 (\vec{J}^{\,0} + \frac{\partial \vec{D}}{\partial t})) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$

$$\nabla \times \nabla \times E = -\mu_0 \varepsilon_0 \, \partial^2 E/\partial t^2 = \nabla(\cancel{\nabla \cdot E}) - \nabla^2 \vec{E} \Rightarrow \nabla^2 E - (\mu_0 \varepsilon_0) \partial^2 E/\partial t^2 = 0$$ wave eq.

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$ compare to $(v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c)$ $\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$ 1D wave eq in material medium

Scalar wave eq in string

$c = 3 \cdot 10^8$ m/s good approximation

Maxwell's Eq $\Rightarrow$ existence of electromagnetic waves. $\frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$

In a medium

ε = permittivity of medium μ = perme~~ability~~ of medium

$v_{medium} = 1/\sqrt{\mu \varepsilon}$ $\vec{\nabla} \cdot \vec{E} = \rho_{total}/\varepsilon = (\rho_f + \rho_b)/\varepsilon$

$\vec{E}, \vec{B}$ field modifies displacement current acts of e^-'s: $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

quartz SiO_2 H apply field material has a dipole moment

induced dipole moment $\vec{p} = q \cdot \vec{r}$ $q_+ \xrightarrow{\vec{r}} q_-$

Sum over macroscopic, ave over volume

10,000+ atoms $\vec{P} = \frac{\sum \vec{p}_i}{V}$ polarization of the medium

dipoles generate fields they modify the displacement vector

$$\vec{P} = (\varepsilon_0 \chi) \vec{E}$$ χ = linear susceptibility of the medium

What is a weak field: compare to atomic field strength ~ H atom use Coulomb's law to get e^-/p^+ field ~ $100 \cdot 10^6$ V/cm

if it is weaker than 100,000,000 V/cm, then it is a weak field

Weak fields have linear response

Phenomena of Strong Field: non-linear responses) $P \propto \chi E + \chi_2 E^2 + \chi_3 E^3 + \cdots$

Second, third order susceptibility

interesting devices use this phenomena

$$\vec{D} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi \vec{E} = \varepsilon_0 (1 + \chi) \vec{E} = \varepsilon \vec{E}$$

$\vec{B} = \mu \vec{H}$ isotropic medium = properties are the same in all directions

$\vec{B} = \mu_0 [1 + X_m] \vec{H}$ X_m is the analog for the magnetic dipoles

Constitutive Relations: $\vec{D} = \varepsilon \vec{E}$ $\varepsilon = \varepsilon_0 (1 + X_e)$ $\vec{\nabla} \cdot \vec{E} = \rho/\varepsilon$

$\vec{B} = \mu \vec{H}$ $\mu = \mu_0 (1 + X_m)$

$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) = \vec{\nabla}(\rho/\varepsilon) \to 0$ for linear, isotropic, homogeneous condition.

dimensionless $\frac{\varepsilon}{\varepsilon_0} = 1 + \chi_e = \underbrace{K_r = \varepsilon_r}_{\text{relative dielectric constant}}$ $\frac{\mu}{\mu_0} = \underbrace{1 + X_m = \mu_r = K_m}_{\text{relative}}$ $v = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{c}{n}$

$n = \sqrt{\mu_r \varepsilon_r}$

[illegible]

Electromagnetic Wave Equations

Thursday

$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Maxwell's equations imply the existence of E/M waves $\quad \vec{\nabla}^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$

$\vec{E} = E_0 \cos(kx - \omega t) = E_0 \cos[(k)(x - vt)]$ $\quad c = \omega/k$

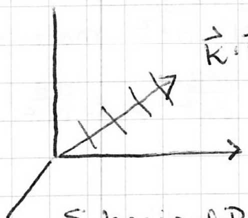
general solution in 1D $\quad$ harmonic solutions, mainly discuss these

Euler Relations

$E = E_0 [\cos(kx - \omega t) + \sin(kx - \omega t)]$ $\quad e^{i\phi} = \cos\phi + i\sin\phi$ $\quad \cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$

$E = E_0 e^{i(kx - \omega t)}$ what we mean is the real part, algebra is easier $\quad \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2}$

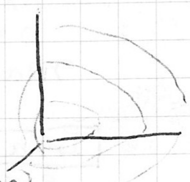
$\boxed{\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}}$ plane wave $\quad$ amplitude remains constant



$\vec{k}\cdot\vec{r} = c$ planes of constant phase

physically: wave w/ well-defined direction continues forever

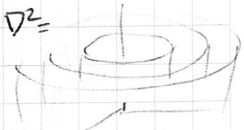
$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$



Spherical Polar Coordinates, surfaces of constant phase are spheres

Spherical Waves $\quad \boxed{\vec{E} = \frac{E_0}{r} \cancel{\cos} e^{i(\vec{k}\cdot\vec{r} - \omega t)}}$ $\quad$ amplitude $\propto 1/r$ $\quad \nabla^2 =$

uniformly illuminated pinhole $\quad (r, \theta, \phi)$

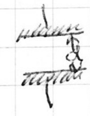


Cylindrical Waves (r, θ, z) $\quad E = \frac{E_0}{\sqrt{r}} e^{i(\vec{k}\cdot\vec{r} - \omega t)}$ $\quad$ larger r

thin slit uniformly illuminated

good approx to plane by collimating

energy flux - the amount of energy per unit area per second



charge capacitor $\quad U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2 = \frac{\varepsilon_0 A}{2d}(E\cdot d)^2 = \frac{1}{2}\varepsilon_0 A d E^2$

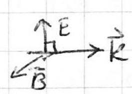
parallel plate capacitor

$u_e = \frac{U}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2 \Rightarrow \frac{1}{2}\varepsilon E^2$ $\quad$ energy density due to electric field

$u_m = \frac{1}{2} B^2/\mu = \frac{1}{2}\mu H^2$ $\quad$ energy density due to magnetic field

Relationships between Electric + Magnetic Fields $\quad |k| = \frac{2\pi}{\lambda}$

$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$ $\quad \vec{B} = \mu\vec{H}$ $\quad \vec{B} = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$



$\vec{\nabla}\cdot\vec{E} = 0 \Rightarrow i\vec{k}\cdot\vec{E} = 0$ $\quad \vec{\nabla}\cdot\vec{B} = 0 \Rightarrow i\vec{k}\cdot\vec{B} = 0$ $\quad$ gauss law

$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \Rightarrow i\vec{k}\times\vec{E} = i\omega\vec{B} \Rightarrow \boxed{\vec{k}\times\vec{E} = \omega\vec{B}}$ $\quad$ faraday's law

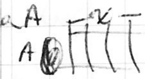
$\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \Rightarrow \boxed{\vec{k}\times\vec{B} = \mu_0\varepsilon_0\,\omega\vec{E}}$ $\quad$ ampere's law, no free current

$E = \frac{\omega}{k}B \Rightarrow E = vB$

In anisotropic medium: $\vec{H}, \vec{D}, \vec{k} \perp$ always but $\vec{E}, \vec{B}, \vec{k}$ only $\perp$ in isotropic medium

$\boxed{E = vB}$ $*$ $\Rightarrow$ total energy density $U_{tot} = u_e + u_m = \varepsilon_0 E^2$ in vacuum $\quad \frac{1}{2\mu} B^2 + \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\varepsilon_0 E^2 =$

Consider a cylinder in which a plane wave is propagating cross-sectional area A



$S = \frac{(U_{total})(x)(cA)}{A}$ = energy flux = $\left(\frac{\text{total energy}}{\text{time} \cdot \text{area}}\right)$ energy propagated through surface. $\varepsilon_0 E^2 c B$

$= \varepsilon_0 E B c^2 = EH$

energy flux due to an E/M wave $\vec{S} = \vec{E} \times \vec{H}$ Poynting's Vector

energy flux $\vec{S} = \vec{E} \times \vec{H}$

Tuesday Feb. 10, 2009 turn-in homework $E = cB$

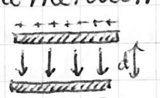
$\vec{S} = \vec{E} \times \vec{H}$ energy per unit area per second carried by E/M wave

$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = c^2 \varepsilon_0 \vec{E} \times \vec{B}$

$E = h\nu \quad p = \frac{h\nu}{c}$ light carries momentum, absorbed $\rightarrow$ forces

Parallel Plate Capacitor $E = \frac{1}{2} C V^2$

$E = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2$



$\vec{F} = -\frac{\partial \vec{E}}{\partial x} = \frac{-\partial}{\partial x}\left(\frac{1}{2}\frac{\varepsilon_0 A E^2 x^2}{x}\right) = -\frac{1}{2}\varepsilon_0 A E^2 = A u_e$

u_e = energy density = $\frac{1}{2}\varepsilon_0 E^2 \quad P = \frac{F_e}{A} = u_e$

Solenoid (Sheet of Current) or $P_m = u_m = \qquad P = U_{total} = \varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$

$**$ $P = \frac{S}{c}$ (pressure) $\quad F = P \cdot A \quad P_v = \frac{S}{c^2}$ (momentum density) $\quad F = \frac{\Delta p}{\Delta t}$ $**$ important to remember

mirror 2x 100% reflection of light pressure = $2S/c$ = energy density

Plane Harmonic Wave $\Psi = A \sin[\phi(x,t)] \qquad \phi = kx - \omega t = k(x - vt)$

$\left|\frac{\partial \phi}{\partial x}\right|_t = |k| \qquad |\omega| = \left|\left(\frac{\partial \phi}{\partial t}\right)_x\right| \qquad v = \frac{-(\partial\phi/\partial t)_x}{(\partial\phi/\partial x)_t} = -\left(\frac{\partial \psi}{\partial t}\right)_x \Big/ \left(\frac{\partial \psi}{\partial x}\right)$

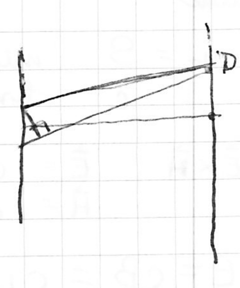
One thing that occurs: Phase Difference if light propagates through a medium

$\Delta\phi = \frac{2\pi}{\lambda}(\underbrace{\Delta n \; x}_{\text{optical path}})$ $\qquad k = \frac{\omega}{v}$

[physical] length

Ⓑ Young's Experiment

$\Delta\phi = \frac{2\pi}{\lambda} n \Delta x$



Ⓐ same distance different refractive indices so the phase velocity is different.

Ⓒ Polarization
Two direction orthogonal electric field
Simple relationship

Closely spaced frequencies + propagations k group velocity

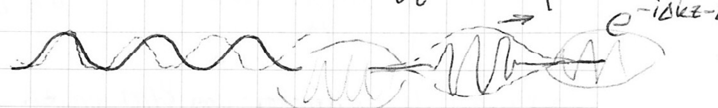
$\boxed{\text{Group Velocity}}$

Superposition of waves close in –

Polarization shows E/M is transverse ⊥ to prop good to keep in mind but usually scalar harmonic wave to describe – but it is a vector

$$U = U_0 e^{i[(k+\Delta k)z - (\omega+\Delta\omega)t]} + U_0 e^{i[(k-\Delta k)z + (\omega+\Delta\omega)t]} = U_0 e^{i(kz-\omega t)} \left(e^{i\Delta kz + \Delta\omega t} + e^{-i\Delta kz - \Delta\omega t} \right)$$

fourier series to construct a pulse



different freq. have different refractive indices, prism (dispersion)

amplitude modulation in radios

just use the identity $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$

$$U = 2U_0 e^{i(kz-\omega t)} \cos(\Delta k z - \Delta\omega t)$$

$\frac{\omega}{k} = v_{phase}$ envelop travels $\frac{\Delta\omega}{\Delta k} = v_{group} = \frac{d\omega}{dk}$

record is a few femto second pulses – lasers can produce 100 fs pulses

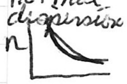
Pure Harmonic Wave is an idealization

when E/M travel in media – dispersion – travel at different speeds, v

n depends on frequency of the light which propagates through it.

digitized data to send 100 ps or less, need different frequency components

bluer frequencies travel more slowly, higher freq. travel slowly ↑n



Repeaters used: {detectors to detect pulses then send another pulse} because of losses, because of dispersion

Have to derive things in next homework

$\boxed{\text{propagation constant } k = n\left(\frac{\omega}{c}\right)}$

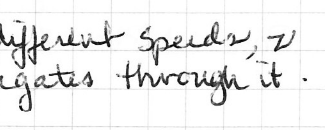
glass, water, something else*: λ_0 is vacuum wavelength

$$\omega = \frac{ck}{n} \quad \frac{d\omega}{dk} = v_{group} = \frac{c}{n} - \frac{ck}{n^2}\frac{dn}{dk} = v_{phase} - \frac{ck}{n^2}\frac{dn}{dk}$$

not very useful relationship

$\boxed{k = \frac{2\pi}{\lambda_{medium}} \quad k_{vac} = \frac{\omega}{c} = \frac{2\pi}{\lambda_{vac}}}$

book $\left(\frac{dn}{d\lambda_0}\right)$ this is useful, the can be measured and quantified



pressure

$P = u_T = \frac{S}{c}$ intensity of light is the same as $I = \langle S \rangle$ time average of Poynting vector

$\vec{S} = \vec{E} \times \vec{H}$ $\vec{E} = \cos(kx - \omega t)$ $\vec{H} = \cos(kx - \omega t)$ $\vec{S} = \vec{E_0} \times \vec{H_0} = \cos^2(kx - \omega t)$

$T = 2\pi/\omega$

$$\langle S \rangle = \frac{\int_0^T \epsilon_0 H_0 \cos^2(kx - \omega t)\, dt}{T}$$

intensity ∝ |ampl|^2

$E = cB = c\mu_0 H_0$

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

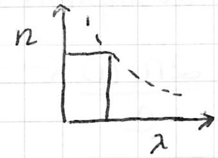
intensity $\langle S \rangle = \frac{1}{2} E_0 H_0 = I = \frac{1}{2} E_0^2 / c\mu_0$ doesn't contribute

12 Feb 09

$u_g = \frac{d\omega}{dk} = \frac{c}{n} - \frac{ck}{n^2}\frac{dn}{dk}$ can rewrite in terms of phase velocity $u_g = u\left(1 - \frac{k}{n}\frac{dn}{dk}\right)$ not useful

$\Rightarrow u_g = u - \lambda \frac{du}{d\lambda}$ useful form $\frac{1}{u_g} = \frac{1}{u} - \frac{\lambda_0}{c}\left(\frac{dn}{d\lambda_0}\right)$

↑ phase velocity



dispersion

Intensity

Phenomenological Model – absorption – forced harmonic oscillator

$\cancel{m}\frac{d^2x}{dt^2} = -\omega_0^2 x$ unforced harmonic oscillator

$\vec{E}$-field is forcing term $\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = \frac{e}{m}F$ $E = E_0 e^{i\omega t}$

trial solution

$\boxed{x = x_0 e^{i\omega t}}$ Substitute $\Rightarrow x = x_0 e^{i\omega t}\frac{E_0}{m}\left(\frac{1}{\omega_0^2 - i\omega\gamma - \omega^2}\right)$

solve for x and you get $x = \frac{eE_0 e^{i\omega t}}{m(\omega_0^2 - \omega^2 - i\gamma\omega)}$ electrons bound by some effective spring constant $\omega_0 = \sqrt{K/m}$

Lorentz Model – for microscopic polarizability

$\vec{P} = N\langle x\rangle$ ← #density ← dipole mmt $= \frac{Ne^2}{m(\omega_0^2 - \omega^2 - i\omega\gamma)}$ $\vec{E} = \vec{P} = \chi\vec{E}$ ← linear susceptibility

polarization

↑ losses from forced oscillation dissipation in the medium

$$\boxed{\chi = \frac{Ne^2}{m(\omega_0^2 - \omega^2 - i\omega\gamma)}}$$

at optical freq. heavy ions can't keep up 10^{14}–10^{15} Hz
at low freq (radio or microwave) the ions can keep up

complex → $\hat{n} = \sqrt{1 + \hat{\chi}} = \sqrt{\hat{\epsilon}_r}$ (if $\mu_r = \mu_0$)

time scale at which field – depends on inertia of atom or molecule

at X-Ray λ $n \to 1$ because the electrons can't keep up with it

like a galvanometer in a fast AC field – the gauge is stuck can't keep up

visible γ is small & real n is real $E = E_0 e^{i(kx - \omega t)}$ optical field

$\hat{n} = n + iK$ ← this k is NOT wavenumber $k = \frac{n\omega}{c}$

if $\hat{n}$ is complex, so is $\hat{k}$, the books have ambiguous definitions symbols

$\hat{n} = n_r + i n_i$ $n = 1.33$ in optical regime water

microwave ovens at GHz $n\uparrow$

dipoles in water, follow field oil/water

Energy $= -\vec{p}\cdot\vec{E}$ OH hydroxyls

water has a large dielectric constant $\epsilon_r(\text{water}) = 80$

~~$E = E_0 e^{i(\omega n_r}$~~ $n = \sqrt{\epsilon_r} = \sqrt{80} \sim 9$ very high

$E = E_0 e^{i\left(\frac{\omega}{c}(n_r + i n_i)x - \omega t\right)}$

$I = \frac{1}{2}(\ )\langle E^2\rangle \propto E_0^2 e^{-2\omega n_i x/c}$

↑ time average

$E = E_0 \underbrace{e^{-(\omega/c)n_i x}}_{}e^{i\left(\frac{\omega}{c}n_i x - \omega t\right)}$

imaginary part gives you an absorbed component

Beer's Law $\boxed{I = I_0 e^{-2n\omega x/c}}$ $I = I_0 e^{-\alpha x}$

... propagating wave,

$I = I_0 e^{-\alpha x} = I_0 e^{-2\eta\omega x/c}$

α = absorption coefficient of the medium [cm^{-1}]

$\alpha = \frac{2n_i\omega}{c}$ ← ω = frequency of light or $\omega = 2\pi\nu$

Concept of introducing damping in the interaction of matter with electromagnetic field – absorption of light

in most metals n_i is large lots of absorption

on the order of microns – it reflects a lot { either n_r large Si or n_i large Al for reflectance

Silicon refractive index in visible – absorps a lot

$n_i = \frac{c \times \alpha}{\omega \times 2}$ $\quad n_r$

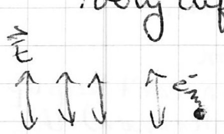
get real!

$\alpha = 10^4\ cm^{-1}$

in cm

$$Si \begin{cases} n_i = \frac{3 \cdot 10^{10}}{3 \cdot 10^{15}} \frac{10^4}{2} \sim .05 \qquad \left(\frac{n-1}{n+1}\right)^2 = R \ ? \\ n_r = 3.6 \end{cases}$$

very different for gold or silver above plasma frequency they are transparent and also for ionosphere FM transmission



$\omega = \sqrt{\frac{k}{m}}$

weak wrt atomic $100\frac{MV}{cm}$ = order

non-linear ~~fields~~ optics causes birefringence.

Polarization $\vec{P} = N(ex) = \chi\vec{E}$

$\frac{100kV}{cm}$ = strong $\vec{E}$ field

if the magnitude of imaginary component is big ⇒ losses are substantial.

Model: Forced Damped HO, assume solution, then use $\vec{P} = N(ex) = \chi\vec{E}$ dipole moment

Forced Damped anharmonic osc. ⇒ non-linear effects quadratic, square components.

$\boxed{\vec{P} = \chi\vec{E} + \chi EE + \chi EEE + \ldots}$ $E = E_0 \cos\omega t$ $\quad EE \propto \cos^2\omega t \quad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

non-linear

manifestations of

frequency doubling

optical rectification, can get most of light to double freq

both Energy + momentum conserved.

forced damped harmonic oscillator

$\epsilon = \epsilon + \frac{i\sigma}{\omega}$ ← Conductivity ⇒ exponential damping only a certain depth

"Skin depth" that E/M will penetrate

$\vec{B} = \mu(\sigma\vec{E} - i\omega\epsilon_r\vec{E}) = -i\omega\epsilon\vec{E}$

connect the conductivity to the complex dielectric, assume $\sigma = 0$ dielectric constant

A/C Conductivity – high frequency

Doppler Effect – Read Chapter in Book + Appendix consider things moving at high velocities – relativistic effects in optics

Discuss { Results, applications of Doppler Effects

$\nu' = \nu\left(\frac{v}{v+u}\right)$ apparent change in frequency v = sound wave velocity, u = receiver velocity

$\nu' = \nu\left(\frac{c}{c+u}\right)$ for light – typically $u \ll c$

$$\nu' = \nu\frac{c}{c(1+u/c)} = \nu(1+u/c)^{-1} \sim \nu(1 - u/c + (u/c)^2 + \dots) \sim \nu(1 - u/c)$$

$\nu' - \nu = \Delta\nu = \frac{u}{c}\nu$ Similarly if the source moves away not much difference

---

Transverse Doppler Shift – only w/ light

Longitudinal Doppler Shift

no transverse Doppler Shift in non-relativistic waves

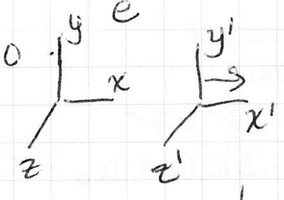
u

$e^{i(kx-\omega t)}$

$\nu' = \nu\left(\frac{\sqrt{1-u/c}}{\sqrt{1+u/c}}\right)$ stars or moving atoms

if $u \ll c$ $\nu' = \nu(1 - u/c + \frac{1}{2}(u/c)^2 + \dots)$

Appendix p. 310



$x = \gamma(x' + ut')$

$y = \gamma(y' + ut')$

$z = \gamma(z' + ut')$

invert this transformation

NOT Galilean transforms!

Lorenze $\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$ Transformation

$\Delta\nu \propto \frac{1}{2}\frac{u^2}{c^2}\nu$ transverse

$\Delta\nu \propto \nu\, u/c$ longitudinal hydrogen Balmer Red line ~600nm $\sim 5 \cdot 10^{14}$ Hz

star moves (1/10) c or (1/1000) c

$$\Delta \nu_{long} = \frac{\nu u}{c} = 5 \cdot 10^{14} \left(\frac{1}{1000}\right) = 5 \cdot 10^{11}$$

experimentally how the component of velocity along obs. line are calculated.

Relative motion ⊥ see Doppler effect

$$\Delta \nu_{transverse} = \frac{1}{2} \nu \left(\frac{u}{c}\right)^2 = \frac{5 \cdot 10^{14}}{2} \left(\frac{1}{1000}\right)^2 \sim 500 \text{MHz}$$ shift measure with interferometer

Doppler effect for sound waves don't observe transverse Doppler effect

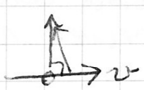
Rain comes down vertically

Car goes $\vec{v}$ seen at angle thru windshield

$$\tan\alpha = \left(\frac{v}{u}\right)$$

(to map the distance of stars)?

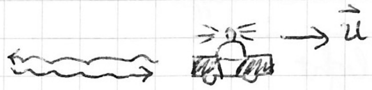
$\tan\theta = v/c$



$\tan\theta = v/c$ ~ microradians aberrations Bradley measured it!

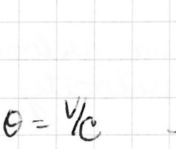
Hubble's Law distance to star ∝ velocity of star $D = Hu$

gravitational Red Shift $\Delta\nu = \nu \frac{u}{c} = 2\nu \frac{u}{c}$



phase of wave which goes and comes back

$$\omega^* = \left(-\frac{\partial\phi}{\partial t}\right)_x$$



$$\Delta\phi = (x + vt)k2 - \omega_0 t$$

there and back

$k = \frac{2\pi}{\lambda}$

mirror is moving with velocity $\vec{v}$

~~$\omega - \omega_0 =$~~

$$\omega = \left(-\frac{\partial\phi}{\partial t}\right)_x \Rightarrow -2kv + \omega_0 = -2\omega_0 \frac{v}{c} + \omega_0 \Rightarrow \omega_0\left(\frac{2v}{c}\right)$$

$\Delta\omega = \omega - \omega_0 =$

NOTE BOOK

Miquelrius

Spectroscopy - started 100 years ago heated up elements (Sodium) white light $\Rightarrow$
absorption (dispersion) spectra Sodium D lines $(D1 - D2) = 6\text{Å}$

Resolution of spectroscopy - broad absorption

Maxwell - Boltzmann velocity distribution in thermal equilibrium $\langle v \rangle = \sqrt{\frac{3kT}{2m}}$?

$$\Delta \nu = \frac{2\sqrt{2\ln 2}}{c}\sqrt{\frac{kT}{m_{atom}}}$$

ν frequency

no broadening FWHM=

laser cooling Bose-Einstein condensation 10^{-8}K
confined to a few 10s of microns + 10^{-8}K observe

$\lambda_b = h/p$ average velocity of atoms is small because

$\lambda_b = h/mv$ λ_b becomes very large - atoms "feel eachother" in quantum mechanical sense - Bose collect in lowest state.

$\boxed{\text{February 24, 2009}}$ Polarization Problems from the book

Problem 2.6, 2.8, 2.9, 2.10, 2.13, 2.11 extremely simple problems

1st midterm March 10, 2009 Polarization Reflection/Refraction.

light is a transverse wave ($\vec{E}\perp$, $\vec{H}\perp$ to $\vec{k}$)
Before Maxwell gave his displacement term came to conclusion
natural minerals around crystal called tourmaline
birefringence slightly shifted images double refraction

The way polarization is achieved - molecules orient pull into plastic
sheet - absorption anisotropic

absorbed more

antenna

intensity

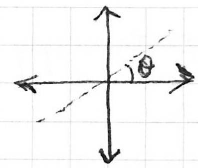
$T = 40\% - 50\%$ for linear polarizer

Degree of Polarization - not "P" # of dipoles per unit volume...

degree of polarization always positive number $= \left|\left(\frac{I_\perp - I_\parallel}{I_\perp + I_\parallel}\right)\right|$ if light is completely along ($\parallel$ or $\perp$) direction $= 1$
if completely random $= 0$

you need to rotate the polarizer to orient along polarization axis
Usually use the orientation of the electric field $\vec{E} = E_0 \exp\{i(\vec{k}\cdot\vec{r} - \omega t)\}$

$\vec{E} = E_0(\hat{\imath} + \hat{\jmath})e^{i(\vec{k}\cdot\vec{r} - \omega t)}$ phase ~~ph~~ factor



actual values of components give polarization

$\vec{E} = E_0\left(\hat{\imath}\cos(kz - \omega t) + \hat{\jmath}\sin(kz - \omega t)\right)$ Right-Circulary Polarized
CCW rotation of $\vec{E}$ as seen from detector of incoming $\vec{k}$

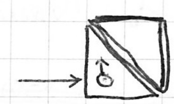
$\vec{E} = E_0\left(\hat{\imath}\cos(kz - \omega t) - \hat{\jmath}\sin(kz - \omega t)\right)$ left-circularly Polarized

Complex Notation $E = E_0\left\{\hat{\imath}\exp i(kz - \omega t) \mp \hat{\jmath}\exp i(kz - \omega t \pm \pi/2)\right\}$

Stick to one notation $\pi \ \pi \ \pi \ \pi \ \pi$

Both the amplitudes can be different — if you have $\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$

anisotropic materials   crystals are cut in a manner



resolve components into the field $\perp$ and $\parallel$ to board

polarization of materials
$\vec{P} = Ne\hat{x}$ e^- is a charged mass isotropic $\vec{P} = \chi\vec{E}$ $\chi = \frac{Ne^2}{m}\frac{1}{(\omega^2 - \omega_0^2 - i\gamma\omega)}$

but the binding of the e^- need not be isotropic, direction
effective spring constant is different in $\perp$ and $\parallel$ directions

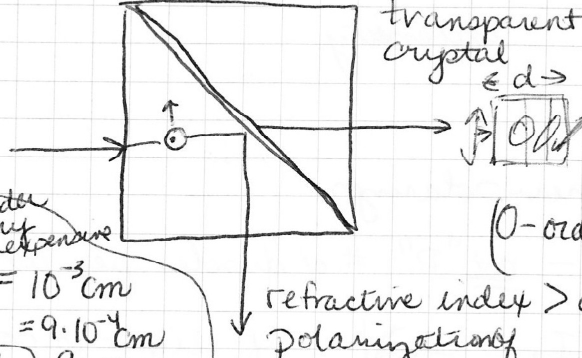
$$\vec{P} = \begin{bmatrix} \chi_{11} & 0 & 0 \\ 0 & \chi_{22} & 0 \\ 0 & 0 & \chi_{33} \end{bmatrix}\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

uniaxial crystals: $\chi_{11} = \chi_{22} = \chi_{33}$ $\varepsilon_y = \varepsilon_x = n_{yy}^2$

$\varepsilon = 1 + \chi$ $\varepsilon_{xx} = \varepsilon_{11} = 1 + \chi_{xx} = 1 + \chi_{11} = n_{yy}^2$

$\varepsilon_z = (1 + \chi_{zz}) = (1 + \chi_{33}) = n_z^2$

(Chose axes right X-Ray crystallography tells how)



transparent crystal   of polarizers glam tompson

d

$5000 \cdot 10^{-10}$
$5 \cdot 10^{-7}$

$n_o = n_z = 1.544$
$n_e = n_y = 1.553$
$.009$

(0-order - insensitive to temperature changes)

~~quarter~~ half wave plate
$d = \frac{\lambda}{2(n_o - n_e)}$
flip polarization $90°$

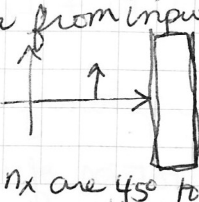
very small "0" order very expensive
$|d| = \frac{5 \cdot 10^{-5}\text{ cm}}{4(.009)} = 10^{-3}\text{ cm} = 9 \cdot 10^{-4}\text{ cm}$
$9\,\mu m$

refractive index > critical angle
Polarization of
can add ~~m dis~~ $n\pi$

Create circularly polarized light out of ~~polarizer~~ from input linearly polarized light

Need
1) equal amplitude
2) $\Delta\phi = \pi/2$ ← want $\pi/2$
$(\pi/2 + 2n\pi)$

~~input~~



Phase Shift from Crystal

$\phi = \frac{2\pi}{\lambda}(nd)$

want

$|d| = \frac{\lambda}{4}\frac{1}{|n_o - n_e|}$ to get circularly polarized light

n_z, n_x are $45°$ to the incident polarization

$\Delta\phi = \frac{2\pi}{\lambda}(n_z - n_y)d \Rightarrow \pi/2$ $(n_o - n_e)$