Homework Set # 3   95.338/538

Due Thursday

Please write legibly and show the steps

use a handbook to get values

1. Make a table of refractive index, n at the wavelength of sodium D-lines (5893 A) and low frequency dielectric(< 1Khz) constant K_E for Water, air , fused silica, Plexiglass (Lucite), and Diamond.

Is the relationship between dielectric constant and refractive index for a nonmagnetic dielectric satisfied in each case? Why do the differences arise? $n = \sqrt{K_e}$

2. Prove the following: a) Curl of **E**= i **kxE**

   b) Divergence of **E**= i **k.E** where **E**= $\mathbf{E_0}\, e^{i(\mathbf{k.r}-\omega t)}$

3. Problem 1.11

4. Problem 2.3

5. a) Problem 2.4 b) 2.5

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1)

	Refractive Index n @ λ=5893Å	Low Freq. Dielectric K_e	$\sqrt{K_e}$
Water	1.33283(@20°C)	~~88~~ 80.4	8.97
Air	1.0003	1.00054	~1
Fused Silica	1.459	3.8	1.95
Lucite	1.495	2.8	1.67
Diamond	2.418	5.5-10	2.3-3.2

because of freq. dep. of K's

Differences arise because

$n = (K K_m)^{1/2}$

most transparent optical media $K_m = 1$

and because $K(\nu)$ and $K_m(\nu)$

2) Prove a) $\vec{\nabla}\times E = i\vec{k}\times\vec{E}$ b) $\vec{\nabla}\cdot\vec{E} = i\vec{k}\cdot\vec{E}$ where $\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

When $\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ since $\vec{\nabla} = \hat{i}\,\partial/\partial x + \hat{j}\,\partial/\partial y + \hat{k}\,\partial/\partial z$

$$\vec{\nabla}\times\vec{E} = \vec{\nabla}\times e^{i(\vec{k}\cdot\vec{r}-\omega t)} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_{0x}e^{()} & E_{0y}e^{()} & E_{0z}e^{()} \end{vmatrix} = (E_{0y} i k_z e^{()} - E_{0z} i k_y e^{()})\hat{i}$$

$$+ (E_{0y} i k_x e^{()} - E_{0x} i k_y e^{()})\hat{k} + (E_{0x} i k_z e^{()} - E_{0z} i k_x e^{()})\hat{j}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ ik_x & ik_y & ik_z \\ E_x & E_y & E_z \end{vmatrix} = i\vec{k}\times\vec{E} \qquad (\vec{\nabla}\to i\vec{k})$$

$$\vec{\nabla}\cdot\vec{E} = ik_x e^{()} E_{0x} + ik_y e^{()} E_{0y} + ik_z e^{()} E_{0z} = i\vec{k}\cdot\vec{E}$$

3) Doppler width of neon line at 100°C.

$T = 100°C = 373K$

$m = 3.34\cdot 10^{-26}$ kg $\qquad u = \sqrt{\frac{kT}{m}} = 392.6\ ms^{-1}$

$k_B = 1.38\cdot 10^{-23}$ J/K

$\lambda = 600$ nm $\qquad \nu = c/\lambda \qquad 2\sqrt{2\ln 2} = 2.355$

$$\Delta\nu = \nu\frac{2\sqrt{2\ln 2}}{c}\sqrt{\frac{kT}{m}} = \frac{2\sqrt{2\ln 2}}{\lambda}\sqrt{\frac{kT}{m}} = \frac{2\sqrt{2\ln 2}\ 392.6ms^{-1}}{600\cdot 10^{-9}m} = 1.59\cdot 10^{9}s^{-1}$$

$$\frac{\Delta\lambda}{\lambda+\Delta\lambda} = \frac{2\sqrt{2\ln 2}}{c}\sqrt{\frac{kT}{m}} \Rightarrow \Delta\lambda = \frac{\lambda\frac{2\sqrt{2\ln 2}}{c}\sqrt{\frac{kT}{m}}}{\left(1 - \frac{2\sqrt{2\ln 2}}{c}\sqrt{\frac{kT}{m}}\right)} = \frac{\frac{600\cdot 10^{-9}(2.355)(392.6)}{3\cdot 10^{8}}}{\left(1 - \frac{2.355(392.6)}{3.0\cdot 10^{8}}\right)}$$

as $\Delta\lambda << \lambda$

$$\Delta\lambda = 1.85\cdot 10^{-12}\,m = 1.85\cdot 10^{-3}\,nm$$

NOTE BOOK

2.3 Peak power of a ruby laser is 100 MW. Beam spot is 10 μm diameter. Find irradiance and amplitude of the electric field of light at the focal point. The index of refraction is 1.

$$I = \frac{\text{Power}}{\text{Area}} = \frac{100 \cdot 10^6\ W}{\pi (5 \cdot 10^{-6}\ m)^2} = 1.27 \cdot 10^{18}\ W m^{-2}$$

$$I = \frac{n|E_0|^2}{2Z_0} \Rightarrow E_0 = \sqrt{\frac{2 Z_0 I}{n}} = \sqrt{2\,(377\Omega)(1.27 \cdot 10^{18}\ W/m^2)} = 3.1 \cdot 10^{10}\ V/m$$

2.4 Show that the average Poynting flux is given by the expression $\frac{1}{2}Re(\vec{E}_0 \times \vec{H}_0^*)$ where $\vec{E}_0$ and $\vec{H}_0$ are the complex field amplitudes of a light wave.

$$\vec{E}_0 = Re(\vec{E}_0) + i\,Im(\vec{E}_0) \qquad \vec{H}_0 = Re(\vec{H}_0) + i\,Im(\vec{H}_0) \qquad i = e^{i\pi/2}$$

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)} \qquad \vec{H} = \vec{H}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)} \qquad \vec{H}_0^* = Re(\vec{H}_0) - i\,Im(\vec{H}_0)$$

$$\vec{H}_0 = Re(\vec{H}_0^*) + i\,Im(\vec{H}_0^*)$$

$$\vec{S} = \vec{E} \times \vec{H} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)} \times \vec{H}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)} = (\vec{E}_0 \times \vec{H}_0) e^{2i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\vec{S} = \left[(Re(\vec{E}_0) + i\,Im(\vec{E}_0)) \times (Re(\vec{H}_0^*) + i\,Im(\vec{H}_0^*))\right] e^{2i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\vec{S} = \left[(Re(\vec{E}_0) + e^{i\pi/2}\,Im(E_0)) \times (Re(\vec{H}_0^*) + e^{i\pi/2}\,Im(\vec{H}_0^*))\right] e^{2i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\vec{S} = e^{2i(\vec{k}\cdot\vec{r} - \omega t)}\left[Re(\vec{E}_0) \times Re(\vec{H}_0^*) + e^{i\pi/2}\left[Im(E_0) \times Re(\vec{H}_0^*) + \cancel{Re(\vec{H}_0^*) \times Re(\vec{E}_0)}\right]^{0} + e^{2i\pi/2}\,Im(\vec{E}_0) \times Im(\vec{H}_0^*)\right]$$

$$\vec{S} = (Re(\vec{E}_0) \times Re(\vec{H}_0^*))\cos^2(\vec{k}\cdot\vec{r} - \omega t) + (Im(\vec{E}_0) \times Im(\vec{H}_0^*))\,i\sin^2(\vec{k}\cdot\vec{r} - \omega t)e^{2i\pi/2}$$

$$\langle \vec{S} \rangle = \underbrace{\langle Re(E_0 \times H_0^*) \rangle}_{\text{constant}} \underbrace{\langle \cos^2(\vec{k}\cdot\vec{r} - \omega t) \rangle}_{1/2} = \tfrac{1}{2} Re(\vec{E}_0 \times \vec{H}_0^*)$$

another (cleaner) derivation →

2.5 The electric vector of a wave is given by the real expression.

$$\vec{E} = E_0[\hat{i}\cos(kz - \omega t) + \hat{j}\,b\cos(kz - \omega t + \phi)]$$

$$\vec{E} = E_0[\hat{i} + \hat{j}\,b\,e^{i\phi}]e^{i(kz - \omega t)}$$

$$\vec{E} = E_0[\hat{i}e^{i(kz - \omega t)} + \hat{j}\,b\,e^{i(kz - \omega t + \phi)}]$$

$$\vec{E} = E_0[\hat{i}\cos(kz - \omega t) + \hat{j}\,b\cos(kz - \omega t + \phi)] + iE_0[\hat{i}\sin(kz - \omega t) + \hat{j}\,b\sin(kz - \omega t + \phi)]$$

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2.4 Show that the average Poynting flux is $\frac{1}{2}\mathrm{Re}(\vec{E}_0 \times \vec{H}_0^*)$

$$\vec{E}(z,t) = \mathrm{Re}[\vec{E}(z)e^{i\omega t}]\hat{x} = \mathrm{Re}[\vec{E}_r(z)e^{i\omega t} + i\vec{E}_i(z)e^{i\omega t}]\hat{x}$$

$$= [\vec{E}_r(z)\cos\omega t - \vec{E}_i(z)\sin\omega t]\hat{x}$$

$$\vec{H}(z,t) = \mathrm{Re}[\vec{H}(z)e^{i\omega t}]\hat{y} = \mathrm{Re}[\vec{H}_r(z)e^{i\omega t} + i\vec{H}_i(z)e^{i\omega t}]\hat{y}$$

$$= [\vec{H}_r(z)\cos\omega t - \vec{H}_i(z)\sin\omega t]\hat{y}$$

$$\vec{P}(z,t) = \vec{E}\times\vec{H} = [\vec{E}_r\vec{H}_r\cos^2\omega t + \vec{E}_i\vec{H}_i\sin^2\omega t - \vec{E}_r\vec{H}_i\cos\omega t\sin\omega t - \vec{E}_i\vec{H}_r\sin\omega t\cos\omega t]\hat{z}$$

$$\langle P\rangle = \frac{1}{T}\int_0^T \vec{P}(z,t)\,dt = \frac{\omega}{2\pi}\int_0^{\omega/2\pi} P\,dt = \frac{\omega}{2\pi}\int_0^{\omega/2\pi}\Big(\underbrace{E_rH_r\cos^2\omega t}_{1/2} + \underbrace{E_iH_i\sin^2\omega t}_{1/2} - \underbrace{E_rH_i\cos\omega t\sin\omega t}_{0} - \underbrace{E_iH_r\sin\omega t\cos\omega t}_{0}\Big)$$

$$\langle P\rangle = \frac{1}{2}(\vec{E}_r\vec{H}_r + \vec{E}_i\vec{H}_i)$$

or

$$\langle\vec{P}(z,t)\rangle = \frac{1}{2}\mathrm{Re}(\vec{E}_0\times\vec{H}_0^*) = \frac{1}{2}\mathrm{Re}[(\vec{E}_r + i\vec{E}_i)\hat{x}\times(\vec{H}_r - i\vec{H}_i)\hat{y}] = \frac{1}{2}(\vec{E}_r\vec{H}_r + \vec{E}_i\vec{H}_i)\hat{z}$$

$$\therefore \quad \langle P\rangle = \frac{1}{2}\mathrm{Re}(\vec{E}_0\times\vec{H}_0^*)$$

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