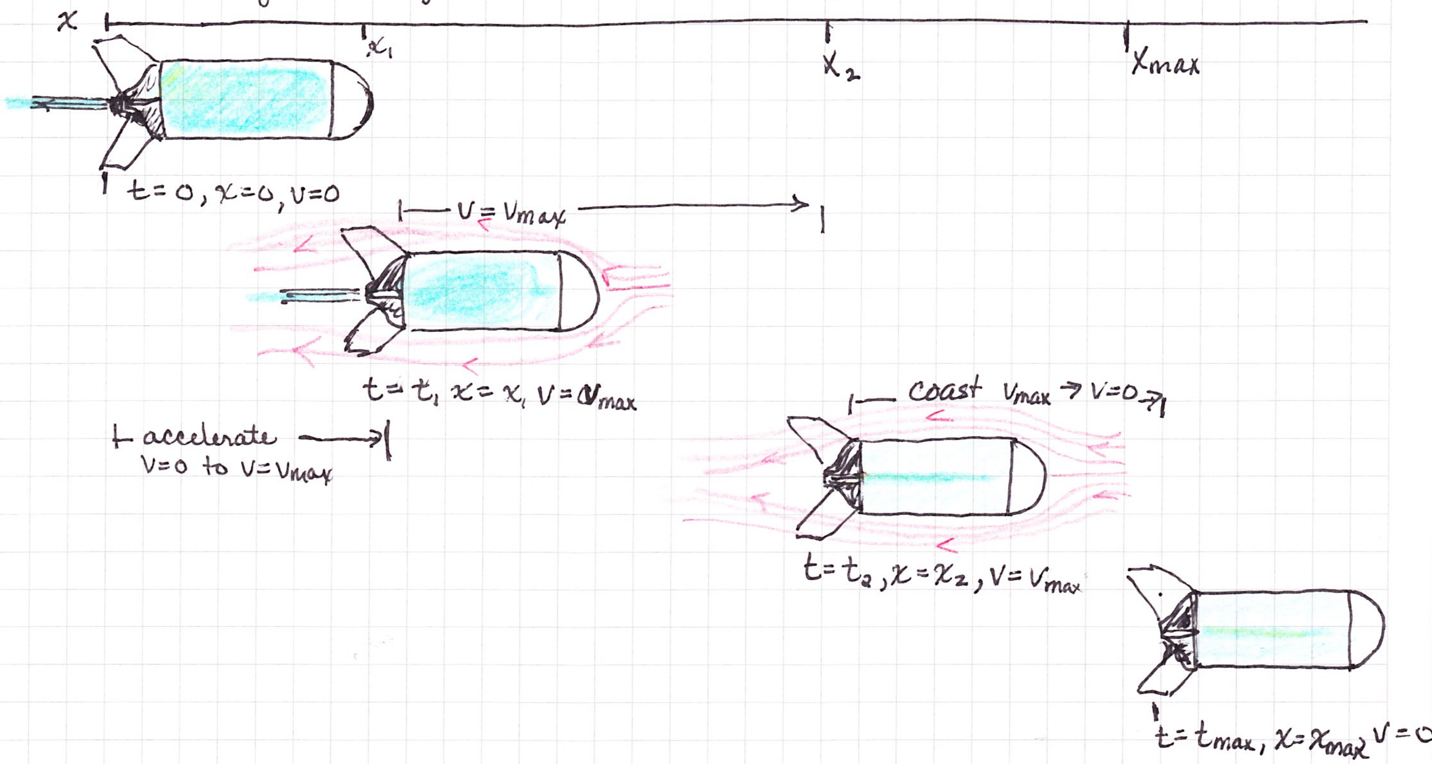
4. Balloon Submarine

a) Briefly describe the basic operation principle.

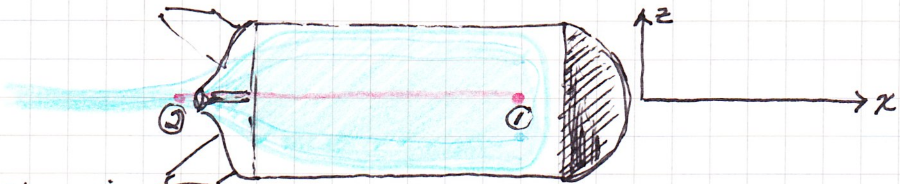
The balloon submarine spews out a jet of water. Thrust, which moves the toy forwards, is the reaction force felt by the submarine. That force, $\Delta P/\Delta t$, is from the product of change of mass of water in the ~~balloon~~ in time and the velocity of the water exiting the balloon (referenced to the exit). The source of energy for forcing the water out of the balloon (and hence propelling the toy forward) is the elastic energy of the balloon. The sources of resistance to forward motion are the friction and drag forces. Drag force includes a viscous term that is proportional to the speed of the toy, and the pressure term that is dependent on the square of the speed. Friction is independent of speed and comes from the intake of water ~~de~~ along the sides. The drag force results in the ~~to~~ toy reaching a terminal (maximum) speed.



b) Determine the fluid exit velocity, u_e, relative to the submarine as a function of the balloon length (filled) L and as a function of time.

quasi-steady: $\frac{\partial \vec{u}_e}{\partial t} = 0$

inviscid: $\frac{\vec{F}}{V_{balloon}} = 0$



Bernoulli Streamline

$$\frac{\partial}{\partial s}\left[\frac{V^2}{2} + \frac{P}{\rho_w} + gz\right] = 0$$

the streamline from ① to ② $ds = dx$ $gz \rightarrow$ constant, ignore

$$\frac{\partial}{\partial x}\left\{\frac{V^2}{2} + \frac{P}{\rho_w}\right\} = 0 \Rightarrow \frac{V^2}{2} + \frac{P}{\rho_w} = \text{constant}$$

$$\frac{V_1^2}{2} + \frac{P_1}{\rho_w} = \frac{V_2^2}{2} + \frac{P_2}{\rho_w}$$

$P_1 = P_2 + \Delta P$ $\Delta P = 100\,kPa - \left(\frac{1m - L}{1m}\right) \cdot 10\,kPa$

$V_1 << V_2$ take $V_1 \sim 0$ $V_2 = u_e$ $\rho_w \sim 10^3\,kg/m^3$

$$\frac{u_e^2}{2} = \frac{1}{\rho_w}(P_1 - P_2)$$

$$u_e = \sqrt{\frac{2}{\rho_w}(\Delta P)} = \sqrt{\frac{2(100\,kPa - [(1m - L)/1m] \cdot 10\,kPa)}{10^3\,kg/m^3}}$$

$\cancel{u_e = \sqrt{200 - (1 - L)10}}$

$$\boxed{\begin{array}{l} u_e = \sqrt{180 + 20L} \quad m/s \quad L \text{ in meters} \quad L > 0 \\ u_e = 0 \quad L = 0,\ t = t_{coast} \end{array}}$$

* The instructions say to find u_e as a function of time but if it is quasi-steady it isn't — L is a function of time thought.

t	u_e
0	14.1 m/s
t_{coast}	13.4 m/s

c) What is the time it takes for the balloon to deflate/evacuate the water inside?

mass of water in balloon $\rho_w V_b \approx \rho_w \pi R^2 L$ $R = .05m$ L is a function of time.

conservation of mass:

$$\frac{\partial}{\partial t}\int_{V_b} \rho_w \, d\tau = -\int_{\substack{\text{exit}\\ \text{area}}} \rho_w u_e r' dr' d\theta'$$

ρ_w = constant $u_e = \sqrt{180 + 20L}$ $L > 0$ uniform over exit $r = .005m$

$$\cancel{\rho_w} \frac{dV_b}{dt} = \cancel{\rho_w} \frac{d}{dt}(\pi R^2 L) = -\pi r^2 u_e \cancel{\rho_w}$$

$$\cancel{\pi} R^2 \frac{dL}{dt} = -\cancel{\pi} r^2 (180 + 20L)^{1/2}$$

$$\frac{dL}{\sqrt{180+20L}} = -\left(\frac{r}{R}\right)^2 dt$$

$$\int \frac{dL}{\sqrt{180+20L}} = -\left(\frac{r}{R}\right)^2 \int dt$$

$$\frac{2\sqrt{180+20L}}{20} = -\left(\frac{r}{R}\right)^2 t + C$$

@ $t=0$ $L = 1m$

$$C = \frac{\sqrt{200}}{10}$$

$$t = \left\{-\frac{\sqrt{180+20L}}{10} + \frac{\sqrt{200}}{10}\right\}\left(\frac{R}{r}\right)^2$$

$$t = \left\{\frac{\sqrt{200}}{10} - \frac{\sqrt{180+20L}}{10}\right\}\left(\frac{R}{r}\right)^2 \qquad \frac{R}{r} = \frac{.05}{.005} = 10$$

$$t = 10\sqrt{200} - 10\sqrt{180+20L}$$

deflated $L \sim 0$

$$t = 10\sqrt{200} - 10\sqrt{180} = 7.26\,s$$

(all quantities in m, s, kg, etc.)

d) Repeat the above two questions assuming that the $\Delta P = 95 KPa = cnst$. Is there a significant difference?

$$u_e = \sqrt{\frac{2\Delta P}{\rho_w}} = \sqrt{\frac{2(95 \cdot 10^3 Pa)}{10^3 kg/m^3}} = \sqrt{190}\ m/s \sim 13.78 m/s$$

$$\frac{\partial}{\partial t}\int \rho_w d\tau = -\int \rho_w u_e r' dr' d\theta'$$

$$R^2 \frac{dL}{dt} = -r^2 \sqrt{190}$$

$$\int dL = -\int \left(\frac{r}{R}\right)^2 \sqrt{190}\, dt$$

$$L = -\left(\frac{r}{R}\right)^2 \sqrt{190}\, t + C \qquad @\ t = 0s,\ L = 1m$$

$$L = -\left(\frac{r}{R}\right)^2 \sqrt{190}\left(\frac{m}{s}\right) t + 1m \qquad R/r = 10$$

when deftated $L \sim 0 \Rightarrow \left(\frac{r}{R}\right)^2 \sqrt{190}\frac{m}{s} t = 1m \Rightarrow t = \left(\frac{R}{r}\right)^2 \frac{1}{\sqrt{190}} = \frac{100}{\sqrt{190}} = 7.25s$

No, there is <u>not</u> a significant difference.

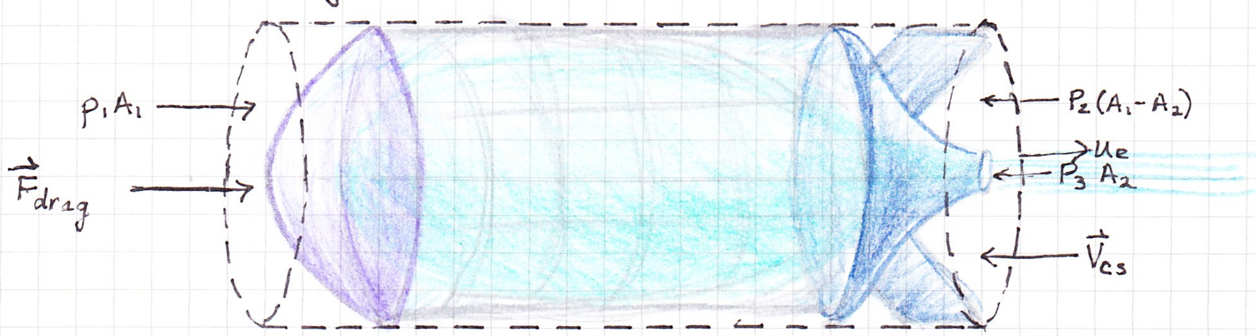
e) Determine the expression for the rate of change of the submarine velocity as a function of time if the fluid inside the ~~submarine~~ balloon is water.

When operating in water, the submarine fills with water the void created by the balloon deflation. The mass of the submarine is constant.

The Volume of the submarine $V_{sub} \sim \pi R^2 L_{max} = \pi (.05)^2 m^3 = 7.85 \cdot 10^{-3} m^3$

Mass of Submarine $M_{sub} \sim \rho_w V_{sub} + M_{structural} \sim \left(\frac{10^3 kg}{m^3}\right)(7.85 \cdot 10^{-3} m^3) + 1kg = 8.85 kg$

Control Volume/Surface is a bounding cylinder around the sub. It is a moving control volume.



drawn backwards $\xleftarrow{+x}$

From the conservation of linear momentum $\vec{V}$ = velocity of control volume.

$$\frac{D}{Dt}\int_{sys} \vec{V}\rho d\tau = \frac{\partial}{\partial t}\int_{cv} \vec{V}\rho d\tau' + \int_{cs} \vec{V}\rho(\vec{u}_e - \vec{V})\cdot\hat{n}\, da = \Sigma \vec{F}$$

$$\frac{\partial}{\partial t}\int_{cv} \vec{V}\rho d\tau = \hat{x}\frac{\partial}{\partial t}\int_{cv} V\rho d\tau = \hat{x}\frac{\partial}{\partial t} V \underbrace{\int_{cv} \rho d\tau} = \hat{x}\frac{\partial}{\partial t} VM = \hat{x}\left(M\dot{V} + \cancel{V\dot{M}}^{\,0}\right) = \hat{x}\, M\dot{V}$$

M = mass of submarine = constant = 8.85kg.

$$\int_{cs} \vec{V}\rho(\vec{u}_e - \vec{V})\cdot\hat{n}\, da = -(u_e + V)V\rho\pi r^2$$

V = +x direction

(cnst) u_e in −x direction $\quad \int_{cs} da = \pi r^2 \quad r = .005\,m \quad \rho = 10^3\,kg/m^3 \quad u_e = 13.78\,m/s$

$\hat{n} = -\hat{x}$

$(\vec{u}_e - \vec{V})\cdot(-\hat{x}) = -(u_e + V)$

$$\Sigma\vec{F} = -P_1 A_1(-\hat{x}) - P_2(A_1 - A_2)\hat{x} - P_3 A_2\hat{x} + \vec{F}_{drag}$$

$P_1 = P_2 = P_3 = 0$ gage because ~~pressure drag is modeled, the viscous drag is ignored.~~ the balloon is surrounded by water filling the void at external pressure.

$\vec{F}_{drag} = -.5\,\frac{kg}{m}\,V^2\hat{x}$ pressure drag is modeled, viscous drag is ignored.

$$\frac{\partial}{\partial t}\int_{cv} \vec{V}\rho d\tau + \int_{cs} \vec{V}\rho(\vec{u}_e - \vec{V})\cdot\hat{n}\, da = \Sigma\vec{F}$$

$$m\dot{V} - (u_e + V)V\rho\pi r^2 = -.5V^2$$

$$m\dot{V} = (u_e\rho\pi r^2)V + (\rho\pi r^2 - .5)V^2$$

$$\left(\frac{m}{u_e\rho\pi r^2}\right)\dot{V} = V + \left\{\frac{\rho\pi r^2 - .5}{u_e\rho\pi r^2}\right\}V^2$$

$$u_e\rho\pi r^2 = (13.78\,m/s)(10^3\,\tfrac{kg}{m^3})\pi(.005m)^2 = 1.08\,kg/s$$

$$\tau\dot{V} = V + aV^2$$

$$\tau = \frac{m}{u_e\rho\pi r^2} = \frac{8.85kg}{1.08kg/s} = 0.122s^{-1}$$

$$\frac{dV}{V(1+aV)} = \frac{dt}{\tau}$$

$$a = \frac{\rho\pi r^2 - .5}{u_e\rho\pi r^2} = \frac{(10^3 kg/m^3)\pi(.005m)^2 - .5}{1.08kg/s} = -.4\,\frac{s}{m}$$

$$\ln\left(\frac{V}{1+aV}\right) = \frac{t}{\tau} + \text{constant}$$

$$\frac{V}{1+aV} = A_0 e^{t/\tau} \rightarrow \frac{V}{1-.4V} = A_0 e^{0.122t}$$

$$\rightarrow \boxed{V = \frac{A_0'}{e^{-.122t} + .4A_0'}}$$

$A_0' = V_0(1 - .4V_0)^{-1}$ where V_0 is the initial velocity from the impulse force of starting this process — the impulse force is not modeled above.

f) Calculate and Plot Velocity during "jet phase".

maximum @ 7.26s = .435 m/s

Assumptions: t=0 after impulse force sets the submarine in motion
The initial velocity (from impulse) is taken as .2 m/s.

Attached matlab code, table, plot.

g) Calculate and Plot the velocity of the submarine during "coast phase."

$$m\dot{V} = (u_e \cancel{\rho \pi r^2}^{\,0}) V + (\cancel{\rho \pi r^2}^{\,0} - .5) V^2$$ $u_e = 0$ $r \to 0$ no exit!

$$m\dot{V} = -.5V^2$$

$$\int_{V_0}^{V} \frac{dV}{V^2} = \frac{-.5}{m} \int_{t_0}^{t} dt$$

$$-\frac{1}{V} + \frac{1}{V_0} = \frac{-.5}{m}(t - t_0)$$

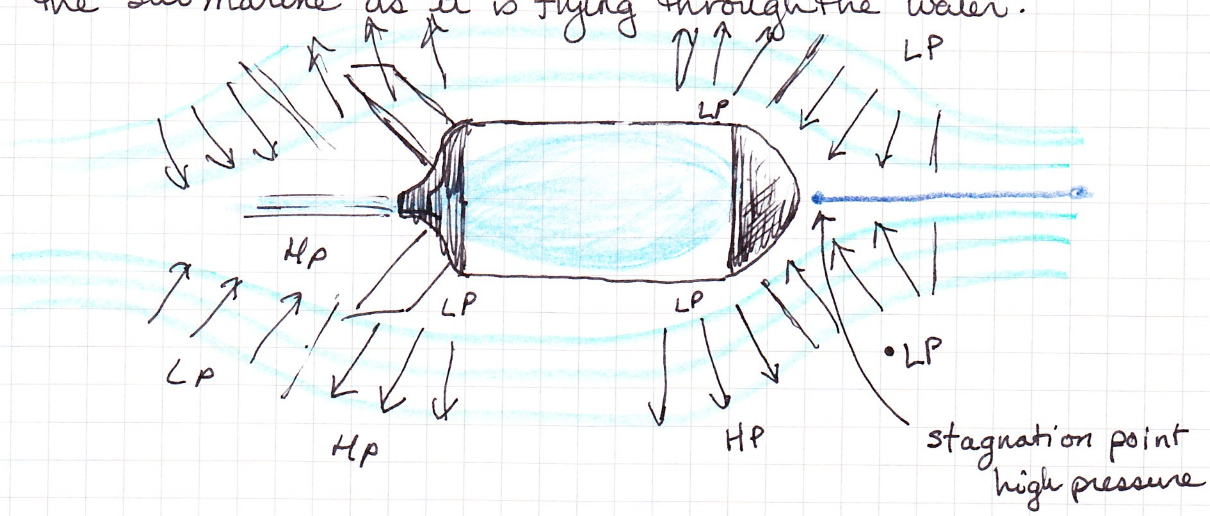
$$V = \frac{V_0}{(.5V_0/m)(t - t_0) + 1}$$

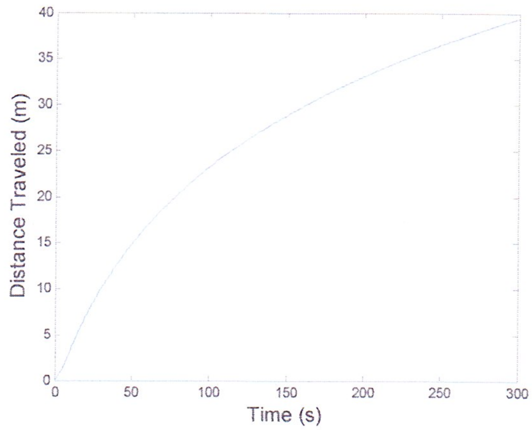
here t_0 = t @ end of jet phase ~7.25 s
$m = 8.85$ kg
$V_0 = .43489$ m/s end of jet phase.

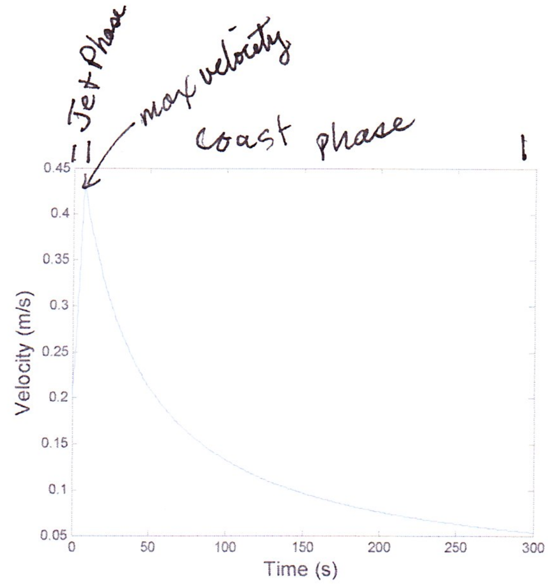
Attached Plot

h) Calculate & plot the distance the submarine travels as a function of ~~time~~.

i) Draw the approximate pressure distribution on the front and back of the submarine as it is flying through the water.







Jet Phase		
T(s)	V (m/s)	X(m)
0.00	0.200000	0.000000
0.25	0.205684	0.050711
0.50	0.211515	0.102861
0.75	0.217496	0.156487
1.00	0.223629	0.211628
1.25	0.229918	0.268321
1.50	0.236366	0.326607
1.75	0.242974	0.386524
2.00	0.249748	0.448114
2.25	0.256688	0.511419
2.50	0.263799	0.576480
2.75	0.271083	0.643340
3.00	0.278543	0.712043
3.25	0.286182	0.782634
3.50	0.294002	0.855157
3.75	0.302008	0.929658
4.00	0.310200	1.006184
4.25	0.318583	1.084782
4.50	0.327158	1.165500
4.75	0.335929	1.248385
5.00	0.344897	1.333489
5.25	0.354066	1.420859
5.50	0.363437	1.510547
5.75	0.373013	1.602603
6.00	0.382797	1.697080
6.25	0.392789	1.794028
6.50	0.402993	1.893501
6.75	0.413410	1.995551
7.00	0.424041	2.100232
7.25	0.434890	2.207599

Coast Phase		
T(s)	V (m/s)	X(m)
8.00	0.427021	2.530815
9.00	0.416961	2.952806
10.00	0.407365	3.364969
20.00	0.331151	7.031538
30.00	0.278960	10.067336
40.00	0.240980	12.657873
50.00	0.212103	14.917211
60.00	0.189406	16.920520
70.00	0.171097	18.719967
80.00	0.156016	20.353236
100.00	0.132634	23.227123
110.00	0.123388	24.506130
120.00	0.115347	25.698911
130.00	0.108290	26.816361
140.00	0.102047	27.867433
150.00	0.096484	28.859572
160.00	0.091497	29.799038
170.00	0.086999	30.691143
180.00	0.082923	31.540434
190.00	0.079212	32.350833
200.00	0.075819	33.125745
210.00	0.072705	33.868150
220.00	0.069836	34.580665
230.00	0.067185	35.265605
240.00	0.064728	35.925023
250.00	0.062445	36.560755
260.00	0.060317	37.174442
269.00	0.058522	37.709137
270.00	0.058329	37.767563
280.00	0.056468	38.341451
290.00	0.054723	38.897315
300.00	0.053081	39.436252

Matlab Code

```
function fluids_midterm_01

V01 = 0.2;M = 8.85;tau =
0.122;a = -.4;
t1 = 0.0:0.25:7.25;
Q = V01/(1 + a*V01);
V1 = Q./(0.4*Q+exp(-tau.*t1));
V02 = V1(length(V1));
t02 = t1(length(t1));
t2 = t02+0.75:1.0:30*t02;
t2 = t02+0.75:1.0:300.0;
R = 0.5*V02/M;
V2 = V02./(R.*(t2-t02) + 1);
t = [t1 t2];
V = [V1 V2];
x = zeros(length(t))
for i=2:length(t)
    x(i) = x(i-1) +
((V(i)+V(i-1))/2.0)*(t(i)-t(i-
1));
end
for i = 1:length(t)
    fprintf(1,'%f %f
%f\n',t(i),V(i),x(i));
end
figure(1);plot(t,V);
xlabel('Time
(s)','fontsize',14);
ylabel('Velocity
(m/s)','fontsize',14);
figure(2);plot(t,x);
xlabel('Time
(s)','fontsize',14);
ylabel('Distance Traveled
(m)','fontsize',14);
```

--M. Noah