3. Balloon Festival Question

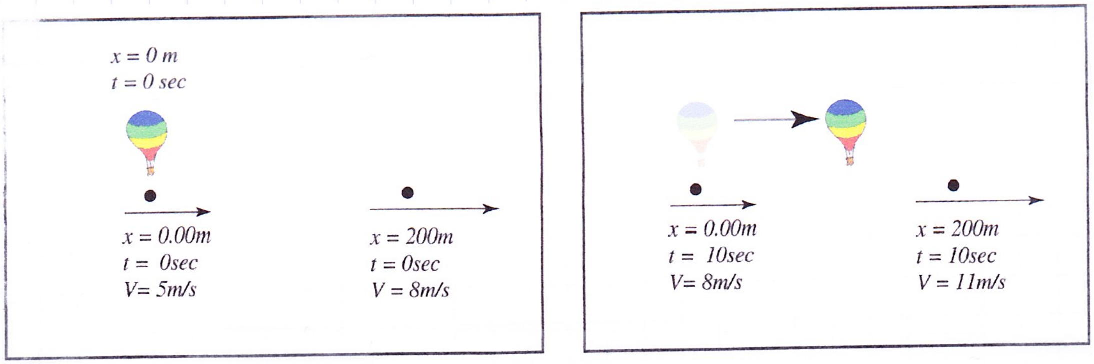

Figure 3: The balloon in a velocity field.

a) Write an expression for the velocity field at any point x (between the two velocity sensors in the fluid at any time t.

Given: Velocity in x-direction only, varies linearly in space and linearly in time. The boundary conditions are specified in Figure 3. Technically this is an overspecified problem, but the B.C.'s do not lead to a contradiction.

$$\vec{v} = c_1 x\,\hat{x} + c_2 t\,\hat{x} + c_3\,\hat{x} \qquad c_1, c_2, c_3 \text{ are constants}$$

$$\vec{v}(x,t) = (c_1 x + c_2 t + c_3)\hat{x}$$

$$\vec{v}(0,0) = 5\,m/s\,\hat{x} = c_3\,\hat{x} \Rightarrow c_3 = 5\,m/s$$

$$\vec{v}(200m,0)\hat{x} = 8\,m/s\,\hat{x} = (c_1(200\,m) + 5\,m/s)\hat{x} \Rightarrow c_1 = \left(\frac{3}{200}\right)s^{-1}$$

$$\vec{v}(0,10s)\hat{x} = 8\,m/s\,\hat{x} = \left[\cancel{\left(\frac{3}{200}s^{-1}\right)}\overset{0}{\cancel{0}} + c_2(10s) + 5\,m/s\right]\hat{x} \Rightarrow c_2 = \left(\frac{3}{10}\right)m/s^2$$

$$\therefore \quad \vec{v}(x,t) = \left[\left(\frac{3}{10}\,ms^{-2}\right)t + \left(\frac{3}{200}\,s^{-1}\right)x + 5\,ms^{-1}\right]\hat{x}$$

b) Write the expression for the local acceleration at a point x.

$$\vec{a}_{local} = \frac{\partial \vec{v}}{\partial t} = \left(\frac{3}{10}\,ms^{-2}\right)\hat{x}$$

c) What is the expression for the ~~total~~ convective acceleration at a point x.

$$\vec{a}_{convective} = \vec{v}\cdot\vec{\nabla}\vec{v} = \left(v\frac{\partial v}{\partial x}\right)\hat{x} = \left[\left(\frac{3}{10}\,ms^{-2}\right)t + \left(\frac{3}{200}\,s^{-1}\right)x + 5\,m/s\right]\left(\frac{3}{200}\,s^{-1}\right)\hat{x}$$

$$= \left[\left(\frac{9}{2000}\,ms^{-3}\right)t + \left(\frac{9}{40{,}000}\,s^{-2}\right)x + \left(\frac{15}{200}\,ms^{-2}\right)\right]\hat{x}$$

d) What is the expression for the total derivative (as a function of x and t) of the velocity (total acceleration)?

$$\vec{a}_{total} = \frac{D\vec{v}}{Dt} = \vec{a}_{local} + \vec{a}_{convective} = \frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{v}$$

$$= \left[\frac{3}{10}\,ms^{-2} + \frac{15}{200}\,ms^{-2} + \left(\frac{9}{2000}\,ms^{-3}\right)t + \left(\frac{9}{40{,}000}\,s^{-2}\right)x\right]\hat{x}$$

$$= \left[\frac{75}{200}\,ms^{-2} + \left(\frac{9}{2000}\,ms^{-3}\right)t + \left(\frac{9}{40{,}000}\,s^{-2}\right)x\right]\hat{x}$$

e) What is the velocity and acceleration of the hot air balloon at time $t = 0$ sec. if it is located at $x = 0$ m?

$$\vec{v}(x,t) = \left[\left(\frac{3}{10}\,ms^{-2}\right)t + \left(\frac{3}{200}\,s^{-1}\right)(x) + 5ms^{-1}\right]\hat{x}$$

$$= \left[\left(\frac{3}{10}\,ms^{-2}\right)0s + \left(\frac{3}{200}\,s^{-1}\right)0m + 5ms^{-1}\right]\hat{x}$$

$$= 5ms^{-1}\,\hat{x}$$

assume balloon moves at the same speed as the wind.

$$\vec{a}(x,t) = \left[\left(\frac{9}{2000}\,ms^{-3}\right)t + \left(\frac{9}{40{,}000}\,s^{-1}\right)x + \left(\frac{15}{200}\,ms^{-2}\right)\right]\hat{x}$$

$$= \left[\left(\frac{9}{2000}\,ms^{-3}\right)0s + \left(\frac{9}{40{,}000}\,s^{-1}\right)0m + \frac{15}{200}\,ms^{-2}\right]\hat{x}$$

$$= \left(\frac{15}{200}\,ms^{-2}\right)\hat{x}$$

$$= \left(\frac{3}{40}\,ms^{-2}\right)\hat{x}$$