.1 A homogeneously broadened laser transition at $\lambda_0 = 10.6\,\mu m$ (CO_2) has the following characteristics:

$A_{21} = 0.34 s^{-1}$ $J_2 = 21$ $J_1 = 20$ $\Delta\nu_h = 1 GHz$

a) What is the stimulated line emission at line center?

$$g_h(\nu) = \frac{\Delta\nu_h}{2\pi[(\nu_0-\nu)^2 + (\Delta\nu_h/2)^2]} \Rightarrow g_h(\nu_0) = \frac{2}{\pi\Delta\nu_h}$$

$$\sigma(\nu_0) = \frac{A_{21}\lambda_0^2}{8\pi n^2} g(\nu_0) = \frac{A_{21}\lambda_0^2}{4\pi^2 n^2 \Delta\nu_h} = \frac{0.34s^{-1}(10.6\cdot10^{-6}m)^2}{4\pi^2(1\cdot10^9 s^{-1})} = 9.7\cdot10^{-18} cm^2$$

b) What must be the population ~~laser transition at λ=~~ inversion density $N_2 - (g_2/g_1)N_1$ to obtain a gain coefficient of 5%/cm? If the lifetime of the upper state is 10 μs and that of the lower state is 0.1 μs what is the saturation intensity?

$$\gamma_0 = 0.05 cm^{-1} = \sigma\Delta N$$

$$\Delta N = \frac{\gamma_0}{\sigma} = \frac{0.05 cm^{-1}}{9.7\cdot10^{-18} cm^2} = 5.15\cdot10^{15} cm^{-3}$$

$$I_s = \frac{h\nu}{\sigma\tau_2} = \frac{hc}{\sigma\tau_2\lambda_0} = \frac{6.626\cdot10^{-34} Js \; 3\cdot10^8 ms^{-1}}{9.7\cdot10^{-18}cm^2 \; 10\mu s \; 10.6\cdot10^{-6}m}$$

$$I_s = 193.3\ W/cm^2$$

$I_{out} - I_{in} = 0.5[\gamma_0 l_g I_s] = 30.3 W/cm^2$ with $\ln\frac{I_0}{I_{in}} + \frac{I_0 - I_{in}}{I_s} = \gamma_0 l_g$

$\Rightarrow \ln\frac{I_0}{I_{in}} + \frac{1}{2}\gamma_0 l_g = \gamma_0 l_g \Rightarrow \ln\left(\frac{I_0}{I_{in}}\right) = \frac{2.704}{2} \Rightarrow \frac{I_0}{I_{in}} = 3.866$

$I_{in}(3.866 - 1) = 30.3 \Rightarrow I_{in} = 10.6 W/cm^2$

$10\log 10 \to 10 dB$ $G = 10$ $10\log(7.9) \to 9.0 dB$ $G' = 7.9$

8.2 An experiment involving a homogeneously broadened optical amplifier is depicted in the diagram.

$I_{in} = 1 W/cm^2$ $G = 10dB$, $I_{in} = 2W/cm^2$ $G = 9dB$

I_{in} → Amplifier → I_{out}

a) What is the small-signal gain ($I_{in} \to 0$) of this amplifier in dB?

$$\ln\frac{I_0}{I_i} + \frac{\bar{g}(\nu)}{I_s}[I_2 - I_1] = \gamma_0(\nu) l_g$$

$$\ln\frac{I_0}{I_i} + \bar{g}(\nu)\frac{I_1}{I_s}\left[\frac{I_2}{I_1} - 1\right] = \gamma_0(\nu) l_g$$

$$\ln G + \bar{g}(\nu)\frac{I_1}{I_s}[G-1] = \gamma_0(\nu) l_g$$

$$\ln G + \frac{I_1}{I_s}[G-1] = \ln G' + \frac{I_1'}{I_s}[G'-1]$$

G = net power gain
γ_0 = small signal gain coefficient
let $\bar{g}(\nu) \approx 1$ line shape near unity at ν_0

$$\ln G - \ln G' = \frac{-I_1[G-1] + I_1'[G'-1]}{I_s}$$

$$I_s = \frac{I_1'[G'-1] - I_1[G-1]}{\ln G/G'} = \frac{-1[10-1] + 2[7.9-1]}{\ln(10/7.9)}\ W/cm^2$$

b) $I_s = 4.8/\ln(10/7.9) = 33.5 W$

$$\gamma_0 l_g = \ln G + \frac{I}{I_s}[G-1] = \ln 10 + \frac{1}{33.5}[10-1] = 2.57$$

a) $G_0 = e^{\gamma_0 l_g} = e^{2.57} = 13.08$

c) What is the maximum power/area that can be extracted?

$(\gamma_0 l_g) I_s = 438 W/cm^2$

d) What must be the input intensity to extract 50% I_s?

$I_{in} = 10.6 W/cm^2$

8.4 The model of Section 8.2 assumed an atomic system with equal degeneracies $g_1 = g_2$. Use the same logic path as used there to find an expression for the small-signal gain coefficient γ_0 and for the saturation intensity I_s for the case where $g_1 \neq g_2$.

steady-state

$$\begin{bmatrix} \left(\frac{1}{\tau_2} + \frac{\sigma I_\nu}{h\nu}\right) & -\frac{g_2}{g_1}\frac{\sigma I_\nu}{h\nu} \\ -\left(\frac{1}{\tau_{21}} + \frac{\sigma I_\nu}{h\nu}\right) & \left(\frac{1}{\tau_1} + \frac{g_2}{g_1}\frac{\sigma I_\nu}{h\nu}\right) \end{bmatrix} \begin{bmatrix} N_2 \\ N_1 \end{bmatrix} = \begin{bmatrix} R_2 \\ R_1 \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$$

$$\alpha N_1 + \beta N_2 = R_1$$
$$\gamma N_1 + \delta N_2 = R_2$$

$$\Rightarrow \frac{\alpha}{\beta} N_1 + N_2 = R_1/\beta$$
$$-\gamma/\delta\, N_1 - N_2 = -R_2/\delta$$
$$\left(\frac{\alpha}{\beta} - \frac{\gamma}{\delta}\right) N_1 = R_1/\beta - R_2/\delta$$

$$\Rightarrow N_1 + \frac{\beta}{\alpha} N_2 = R_1/\alpha$$
$$- N_1 - \delta/\gamma\, N_2 = -R_2/\gamma$$

$$\beta\delta(\alpha\delta - \gamma\beta) N_1 = \beta\delta(\delta R_1 - \beta R_2)$$

$$\left(\frac{\beta}{\alpha} - \frac{\delta}{\gamma}\right) N_2 = \left(\frac{R_1}{\alpha} - \frac{R_2}{\gamma}\right)$$

$$N_1 = \frac{\delta R_1 - \beta R_2}{(\alpha\delta - \gamma\beta)}$$

$$\alpha\gamma(\beta\gamma - \alpha\delta) N_2 = \alpha\gamma(\gamma R_1 - \alpha R_2)$$

$$N_2 = \frac{-\gamma R_1 + \alpha R_2}{-\beta\gamma + \alpha\delta)} \qquad N_1 = \frac{-\beta R_2 + \delta R_1}{-\gamma\beta + \alpha\delta)}$$

$$\Delta \equiv \beta\gamma + \alpha\delta = -\frac{g_2}{g_1}\frac{\sigma I_\nu}{h\nu}\left(\frac{1}{\tau_{21}} + \frac{\sigma I_\nu}{h\nu}\right) + \left(\frac{1}{\tau_2} + \frac{\sigma I_\nu}{h\nu}\right)\left(\frac{1}{\tau_1} + \frac{g_2}{g_1}\frac{\sigma I_\nu}{h\nu}\right)$$

$$= \frac{1}{\tau_1\tau_2}\left[1 + \frac{\sigma I_\nu}{h\nu}\left(\tau_2\left(1 - \frac{g_2}{g_1}\frac{\tau_1}{\tau_{21}}\right) + \tau_1\right)\right]$$

$$N_2 = \left(\frac{1}{\tau_2} + \frac{\sigma I_\nu}{h\nu}\right) R_2 + ($$