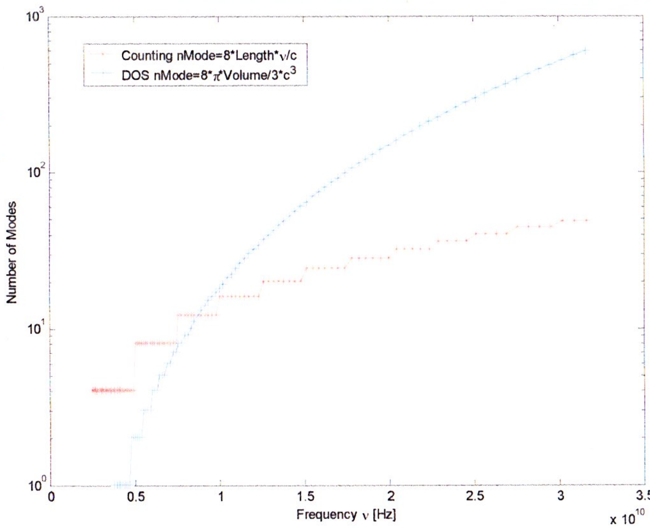
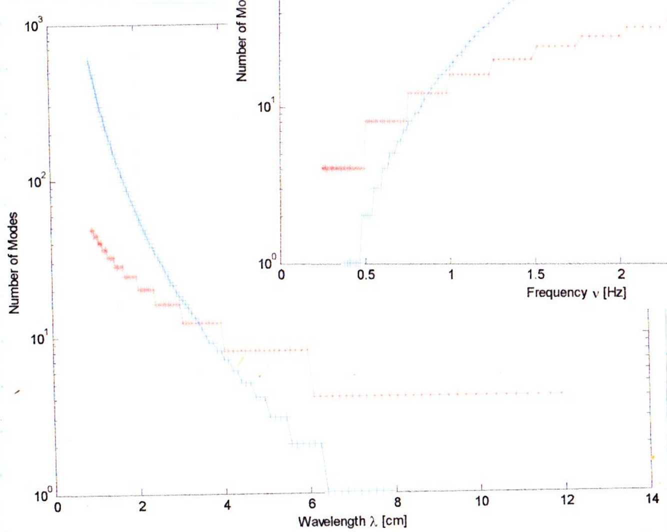
7.1

$$\frac{\Delta\lambda}{\lambda} = 2\sqrt{2\ln 2\,\frac{kT}{m_0c^2}} \qquad \frac{N_m}{V} = \frac{8\pi}{\lambda^3}\frac{\Delta\lambda}{\lambda}$$

$$N_m = \frac{8\pi}{\lambda^3}\frac{\Delta\lambda}{\lambda} V = \frac{16\pi}{\lambda^3}\sqrt{\frac{2\ln 2\, kT}{m_0c^2}}\, L^3$$



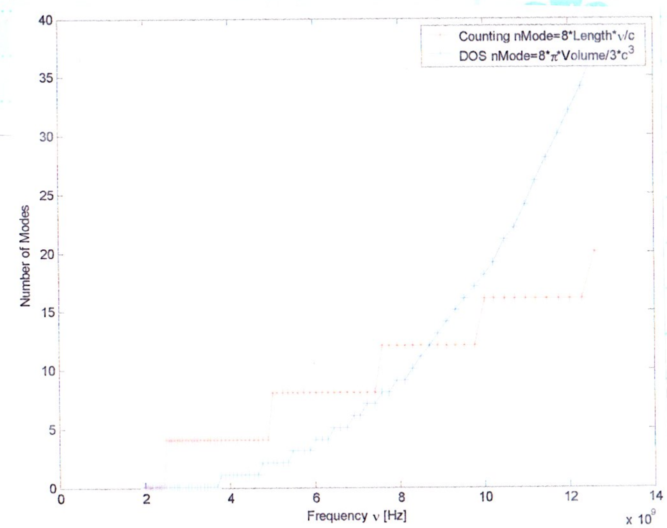


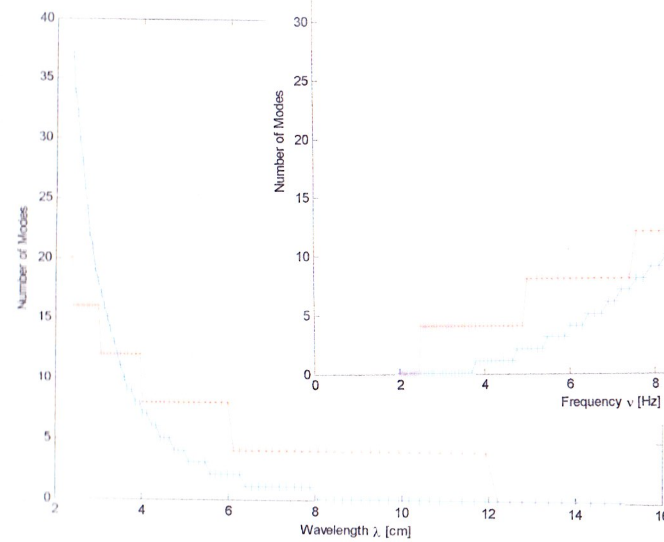

```
function Lasers_Ch7_Pr1a

c=3e10;Volume=2.0*5.0*6.0;d=6;

dv = 0.01;p=9.3:dv:10.1;v=10.^p;
n2=floor((2*d/c)*v);
A = ((8*pi*Volume)/(3*c^3));
n= floor(A*v.^3);

figure(2);hold off;
plot(v,n2*4,'-*r');hold on;
plot(v,n,'-+b');hold on;
xlabel('Frequency \nu [Hz]');ylabel('Number of Modes');
legend(['Counting nMode=8*Length*\nu/c';
    'DOS nMode=8*\pi*Volume/3*c^3 ']);
figure(3);hold off;
plot(c./v,n2*4,'-*r');hold on;
plot(c./v,n,'-+b');hold on;
xlabel('Wavelength \lambda [cm]');ylabel('Number of Modes');
legend(['Counting nMode=8*Length*\nu/c';
    'DOS nMode=8*\pi*Volume/3*c^3 ']);
```





7.2 Show that the ratio of stimulated emission to spontaneous emission is equal to the number of photons per mode. (Fowles)

$$\left.\frac{dN_2}{dt}\right|_{\text{spontaneous emission}} = -A_{21}N_2$$

$$\left.\frac{dN_1}{dt}\right|_{\text{absorption}} = N_1B_{12}\rho(\nu) \qquad \left.\frac{dN_2}{dt}\right|_{\text{stimulated emission}} = -B_{21}N_2\rho(\nu)$$

where $\rho(\nu)$ is the energy density of radiation of frequency ν

$$\nu = \frac{(E_2 - E_1)}{h} \qquad \frac{N_2}{N_1} = \frac{e^{-E_2/KT}}{e^{-E_1/KT}} \text{ equilibrium}$$

equilibrium conditions

$$N_2A_{21} + N_2B_{21}\rho(\nu) = N_1B_{12}\rho(\nu) \quad \text{and } B_{12} = B_{21}$$

solve for $\rho(\nu)$

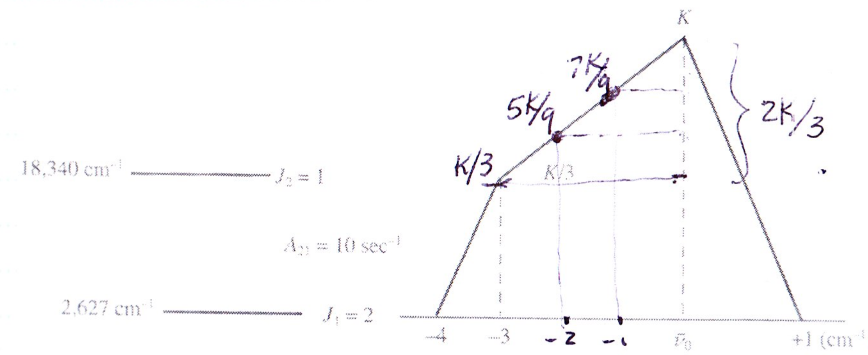
Planck radiation

$$\rho(\nu) = \frac{A_{21}}{B_{21}}\frac{1}{(B_{12}/B_{21})e^{h\nu/KT}-1} = \frac{8\pi h\nu^3}{c^3}\left(\frac{1}{e^{h\nu/KT}-1}\right)$$

$$\therefore \frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

$$\frac{\text{stimulated}}{\text{spontaneous emission}} = \frac{B_{21}\rho(\nu)}{A_{21}} = \frac{1}{e^{h\nu/KT}-1}$$

which is the number of photons per mode



7.3 Spontaneous emission profile ↑

a) What is the stimulated emission cross-section?

$$\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c} \Rightarrow g(\bar{\nu}) = c\,g(\nu) \Rightarrow g(\nu) = c^{-1}g(\bar{\nu})$$

(7.4.7) $$\sigma(\nu) = \frac{A_{21}\lambda^2}{8\pi n^2}g(\nu) = \frac{A_{21}\lambda^2(g(\bar{\nu}))}{8\pi n^2} \qquad g(\bar{\nu}) \text{ in cm}$$

$g(\bar{\nu})$ is piecewise linear

$$\cancel{\sigma(\bar{\nu}) = \frac{(10s^{-1})\left(\frac{1}{2627} - \frac{1}{18340}\right)^2 cm^2}{8\pi(1)^2(3\cdot10^{10}cm\,s^{-1})}}\; g(\bar{\nu})$$

$$\int_0^\infty g(\nu)d\nu = 1$$

$$\cancel{\sigma(\bar{\nu}) = 1.4\cdot10^{-18}}\,g(\bar{\nu}) \qquad \frac{K}{3} + \frac{5K}{9} + \frac{7K}{9} + K = 1 \Rightarrow (3+5+7+9)K = 9 \Rightarrow K = (9/24)$$

$$\sigma(\bar{\nu}) = \frac{(10s^{-1})\left(\frac{1}{18340-2627}\right)^2 cm^2}{8\pi}\,g(\bar{\nu}) \qquad g(\bar{\nu}) = \left(\frac{9}{24}\right)\left(\frac{9-2(\bar{\nu}-\bar{\nu}_0)}{9}\right)$$

$$\sigma(\bar{\nu}) = 1.6\cdot10^{-9}cm^2\left(\frac{9-2(\bar{\nu}-\bar{\nu}_0)}{24}\right)$$

$$\sigma(\bar{\nu}_0) = 6\cdot10^{-10}cm^2$$

b) What is the absorption cross section?

$$\sigma_{abs}(\nu) = \left(\frac{g_2}{g_1}\right) \sigma_{stim}(\nu) \qquad (7.5.4)$$

for a simple atom, this degeneracy factor is related to the total angular momentum:

$$g_2 = 2J_2 + 1 = 3 \qquad g_1 = 2J_1 + 1 = 5$$

$$\sigma_{abs}(\nu) = \frac{3}{5}\sigma_{stim}(\nu)$$

7.5 Consider a transition of 5000Å with a width of 1Å, a cavity of $2cm^3$ in volume and let $n=1$.

a) Convert this wavelength interval (1Å) to frequency.

$$\nu = c/\lambda \quad d\nu = (c/\lambda^2)d\lambda = \frac{(3\cdot10^8 ms^{-1})(1Å)}{(5000Å)^2 10^{-10} m/Å} = \cancel{6\cdot10^{14} Hz} = 120GHz$$

$$\cancel{6\cdot10^5 GHz}$$

$$d\bar{\nu} = d\nu/c = \frac{1.2\cdot10^{11}\ \cancel{6\cdot10^{14}} Hz}{3\cdot10^{10} cms^{-1}} = \cancel{20{,}000} \; 4\,cm^{-1}$$

b) How many electromagnetic modes exist in this frequency band for this cavity?

$$\nu = \frac{c}{\lambda} = \frac{3\cdot10^8 ms^{-1}}{5000\cdot10^{-10} m} = 6\cdot10^{14} Hz$$

the number of electromagnetic modes between ϕ and ν per unit volume of a cavity

$$N(\nu) = \frac{8\pi\nu^3}{3(c/n)^3} = \qquad n=1$$

index of refract

number of modes

$$N = N(\nu)V = \frac{8\pi V}{3\lambda^3} = \frac{8\pi(2cm^3)}{3(5000\cdot10^{-10}m)^3}\left(\frac{m}{10^2 cm}\right)^3$$

$$N = 1{,}34\cdot10^{11} \qquad N_m = N\frac{\Delta\lambda}{\lambda} = 2{,}68\cdot10^7$$

$$N_m = \frac{8\pi}{\lambda^3}\frac{\Delta\lambda}{\lambda}V = \frac{16\pi}{\lambda^3}\sqrt{\frac{2\ln 2kT}{m_0c^2}}\,L^3$$

c) Cylindrical cavity, $1cm^2 \times 20cm$. How many $TEM_{0,0,q}$ modes?

$$L = \frac{\lambda}{2q} \Rightarrow q = (\lambda/2L)^{-1} = \left(\frac{5000\cdot10^{-10}m}{2(20\cdot10^{-2}m)}\right)^{-1} = 8\cdot10^5$$

d) Probability of spontaneous photon in $TEM_{0,0,q}$ mode: $P = \frac{8\cdot10^5}{2.68\cdot10^7} \approx 3\%$

e) If $A = 10^7\,\cancel{cm}s^{-1}$ what is the stimulated cross-section?

$$\sigma = \frac{A\lambda^2 g(\nu)}{8\pi n^2} = \frac{10^7 s^{-1}(5000\cdot10^{-10}m)^2(.03)s}{8\pi} = 3\cdot10^{-9}m^2 =$$

$$= 3\cdot10^{-13}cm^2$$

7.6 Evaluate the Doppler widths of the following He-Ne transition

a) $3s_2 - 2p_4$ at $\lambda_0 = 6328 Å$ red (primary)

b) $2s_2 - 2p_4$ at $\lambda_0 = 1.1523 \mu m$

c) $3s_2 - 3p_4$ at $\lambda_0 = 3.39 \mu m$

Ne = 20.18 amu He = 4amu

T = 300K M ≈ ~~2x amu~~ 20amu p.196 (also 22am)

$k_BT \sim 40eV$ @ room temp, 1AMU = $1.67252 \cdot 10^{-27}$ kg

$$\Delta\nu_D = \left(\frac{8 k_B T \ln 2}{Mc^2}\right)^{1/2} \nu_0 \qquad (7.6.17)$$

$$\nu_0 = c/\lambda_0 \qquad \Delta\lambda_D = \lambda_0^2 \Delta\nu_D / c$$

answer

```
function Lasers_Ch7_Pr6
% Compute doppler broadening of Neon Laser lines
% outputs:
% Neon Doppler Multiplier (T=300K)=2.76209e-006
%   λ0     ν0        Δν_D     Δλ_D
% L0 (A) f0 (GHz) dfD (GHz) dLD (A)
%   6328  4740834    13.09  0.0175
%  11523  2603489     7.19  0.0318
%  33900   884956     2.44  0.0936
%
Lamda0=[6328e-10,1.1523e-6,3.39e-6];
c=3e8;T=300;v0=c./Lamda0;
kB=1.3806503e-23;% m2 kg s-2 K-1
M_Neon=20.0*1.67252e-27; % amu kg/amu
A=sqrt(8.0*kB*T*log(2.0)/(M_Neon*c*c))
vD = A.*v0;
LamdaD=(Lamda0.*Lamda0.*vD./c)*1e10; %Angstroms
fprintf(1,'Neon Doppler Multiplier (T=300K)=%g\n',A);
fprintf(1,'L0 (A) f0 (GHz) dfD (GHz) dLD (Ang)\n');
for i=1:3
fprintf(1,'%6.0f\t%7.0f\t%5.2f\t%6.4f\n', ...
   Lamda0(i)*1e10,v0(i)*10e-9,vD(i)*10e-9,LamdaD(i));
end
```

7.7 Note that many of the HeNe laser transitions share a common upper level (the 1st named state) or a common lower level (the 2nd) in 7.6. Construct energy level diagram

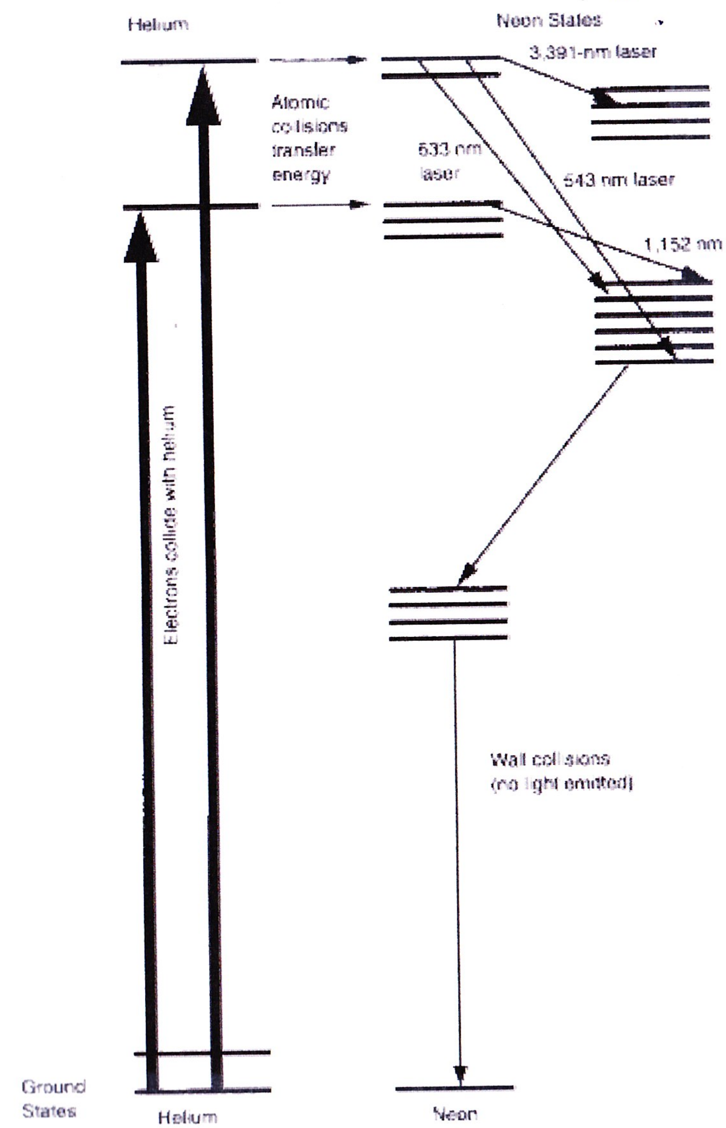


Figure 16.4 Energy levels in HeNe laser: wide arrows show laser transitions (simplified and not to scale)

.8 Reformulate 7.6.15 using the following substitutions

$$w = \frac{2(\nu-\nu_0)}{\Delta\nu_D}\sqrt{\ln 2} \quad a = \frac{\Delta\nu_n}{\Delta\nu_D}\sqrt{\ln 2} \quad y = \frac{2\nu_z}{c}\nu_0\frac{\sqrt{\ln 2}}{\Delta\nu_D}$$

$$g(\nu) = \left(\frac{M}{2\pi kT}\right)^{1/2}\int_{-\infty}^{+\infty}\frac{\Delta\nu_n}{2\pi[(\nu-\nu_0-\nu_0\nu_z/c)^2+(\Delta\nu_n/2)^2]}e^{-M\nu_z^2/2kT}\,d\nu_z$$

$$(\Delta\nu_n/2)^2 = a^2\cancel{\ln 2}(\Delta\nu_D)^2/2^2(\ln 2) = (\Delta\nu_D/2\sqrt{\ln 2})^2 a^2$$

$$\nu-\nu_0 = \frac{\Delta\nu_D\, w}{2\sqrt{\ln 2}} \qquad \cancel{e^{-y^2} = e^{-4\ln 2\,\nu_z^2/c^2\Delta\nu_D^2}}\ e^{-\nu_z^2}$$

$$\cancel{e^{-M\nu_z^2/2kT}} = e^{-\nu_z^2}e$$

$$\frac{Mc^2}{2kT} = \ln 2\,\frac{\nu_0^2}{\Delta\nu_D^2} \Rightarrow \frac{M\nu_z^2}{2kT} = \frac{\nu_z^2\nu_0^2}{c^2\Delta\nu_D^2}\ln 2 = \frac{y^2}{4}$$

$$\Delta\nu_n\, d\nu_z = \left(\frac{a\,\Delta\nu_D}{\sqrt{\ln 2}}\right)dy\left(\frac{c\,\Delta\nu_D}{2\nu_0\sqrt{\ln 2}}\right) = \frac{ac(\Delta\nu_D)^2}{2\nu_0\ln 2}\,dy$$

$$(w-y)^2 = \left(\frac{2\sqrt{\ln 2}}{\Delta\nu_D}\right)^2((\nu-\nu_0)^2 - \nu_z\nu_0/c)^2$$

$$\Rightarrow ((\nu-\nu_0)^2 - \nu_z\nu_0/c)^2 = \left(\frac{\Delta\nu_D}{2\sqrt{\ln 2}}\right)^2(w-y)^2 \quad ①$$

$$(\Delta\nu_n/2)^2 = \left(\frac{\Delta\nu_D}{2\sqrt{\ln 2}}\right)^2 a^2 \quad ②$$

$$\left(\frac{M}{2kT}\right)^{1/2} = \frac{\sqrt{\ln 2}\,\nu_0}{c\,\Delta\nu_D} \quad ③ \qquad e^{-M\nu_z^2/2kT} = e^{-y^2/4} \quad ④$$

$$\Delta\nu_n\, d\nu_z = \frac{ac(\Delta\nu_D)^2}{2\nu_0\ln 2}\,dy$$

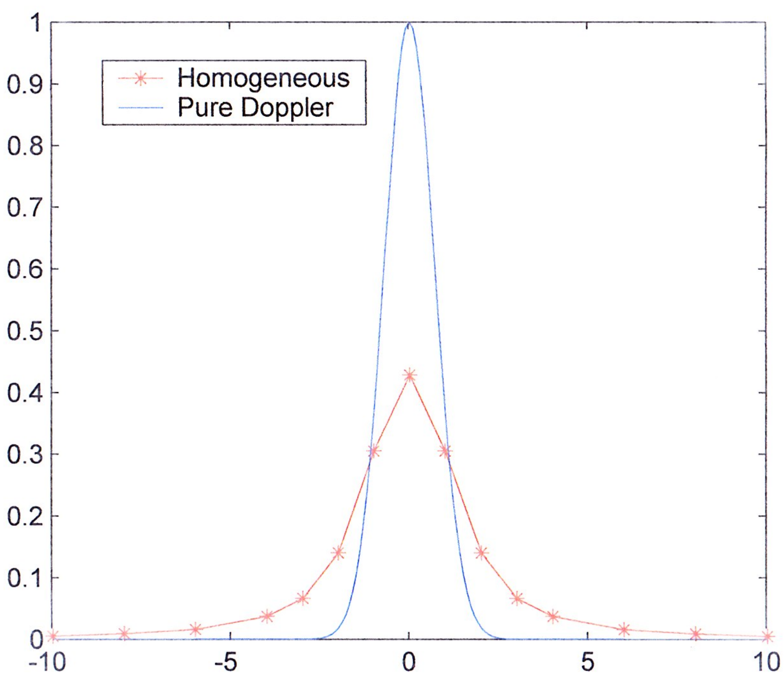
Substitute them

$$g(\nu) = \frac{1}{\sqrt{\pi}}\frac{\sqrt{\ln 2}\,\nu_0}{c\,\Delta\nu_D}\int_{-\infty}^{+\infty}\frac{e^{-y^2}\,dy}{2\pi\left(\frac{\Delta\nu_D}{2\sqrt{\ln 2}}\right)^2[a^2+(w-y)^2]}.$$

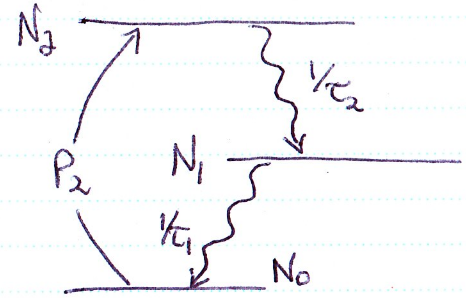
$$g(\nu) =$$

7.9 Plot combined Doppler + Homogeneous and Pure Doppler line profiles.

```
function Lasers_Ch7_Pr9
w=[-10,-8,-6,-4,-3,-2,-1,0,1,2,3,4,6,8,10];
g_bar=[0.005,0.009,0.016,0.037,0.066,0.140,0.305,...
      0.428,0.305,0.140,0.066,0.037,0.016,0.009,0.005];
g=exp(-w.^2);hold off;plot(w,g_bar,'-*r');hold on;
v=-10:0.1:10;g=exp(-v.^2);plot(v,g);
legend(['Homogeneous '; 'Pure Doppler']);
```



7.10 Pump P_2 excites atoms from state ϕ to state 2, nothing to state 1. Assume state $\emptyset$ is not depleted to any significant extent for any time: $dN_0/dt = \phi$.
Use simple decay route with $\tau_2 = 1$ $\tau_1 = 2\,\mu s$
$P_2 = 10^{20}\,cm^{-3}s^{-1}$. Use symbols/values for a, b, c, d.



Follows Case 1 on p214 using $\tau_{21} = \tau_2$ (no branching)
a) What are the rates for states 2 and 1?
b) Give an expression for densities as function of time

$$\frac{dN_2}{dt} = P_{21} - \frac{N_2}{\tau_2} \qquad \text{let } u = P_{21} - N_2/\tau_2$$

$$du = -dN_2/\tau_2$$

$$\frac{dN_1}{dt} + \frac{N_1}{\tau_1} = \frac{N_2}{\tau_2} \qquad -\frac{\tau_2 du}{u} = dt$$

$$u = e^{-t/\tau_2}$$

$$P_{21} - N_2/\tau_2 = e^{-t/\tau_2}$$

$$\boxed{N_2 = \tau_2 (P_2)(1 - e^{-t/\tau_2})}$$

$$\frac{dN_1}{dt} + \frac{N_1}{\tau_1} = P_2(1 - e^{-t/\tau_2})$$

$$\frac{d}{dt}\{N_1 e^{t/\tau_1}\} = \frac{N_2 e^{t/\tau_1}}{\tau_2}$$

$$N_1 = \frac{e^{-t/\tau_1}}{\tau_2}\int_0^t N_2 e^{t/\tau_1}\,dt$$

$$N_1 = P_{21}\tau_1 \left\{ 1 + \frac{(\tau_1/\tau_2)e^{-t/\tau_1}}{(1-\tau_1/\tau_2)} - \frac{e^{-t/\tau_2}}{(1-\tau_1/\tau_2)} \right\}$$

7.10 c) over what time interval δt is the population difference $N_2 - N_1 > 0$?

$$N_1 = N_2$$

$$P_2\tau_1\left\{1 + \frac{\tau_1/\tau_2\, e^{-t/\tau_1}}{(1-\tau_1/\tau_2)} - \frac{e^{-t/\tau_2}}{(1-\tau_1/\tau_2)}\right\} = P_2\tau_2(1-e^{-t/\tau_2})$$

$$\frac{\tau_1}{\tau_2} + \frac{(\tau_1/\tau_2)^2 e^{-t/\tau_1}}{(1-\tau_1/\tau_2)} - \frac{(\tau_1/\tau_2)e^{-t/\tau_2}}{(1-\tau_1/\tau_2)} + e^{-t/\tau_2} - 1 = 0$$

$$\tau_1/\tau_2 = 2$$

$$2 + \frac{4e^{-t/\tau_1}}{(-1)} - \frac{2e^{-t/\tau_2}}{(-1)} + e^{-t/\tau_2} - 1 = 0$$

$$1 - 4e^{-t/\tau_1} + 3e^{-t/\tau_2} = 0$$

$$1 - 4e^{-t/2} + 3e^{-t} = 0$$

let $x = -t/2$

$$3e^{2x} - 4e^x + 1 = 0$$

$$(3e^x - 1)(e^x - 1) = 0$$

$e^x = 1$ $3e^x = 1$

$x = 0$ $e^x = 1/3$

$x = \ln(1/3)$ $\rightarrow -t/2 = \ln(1/3)$

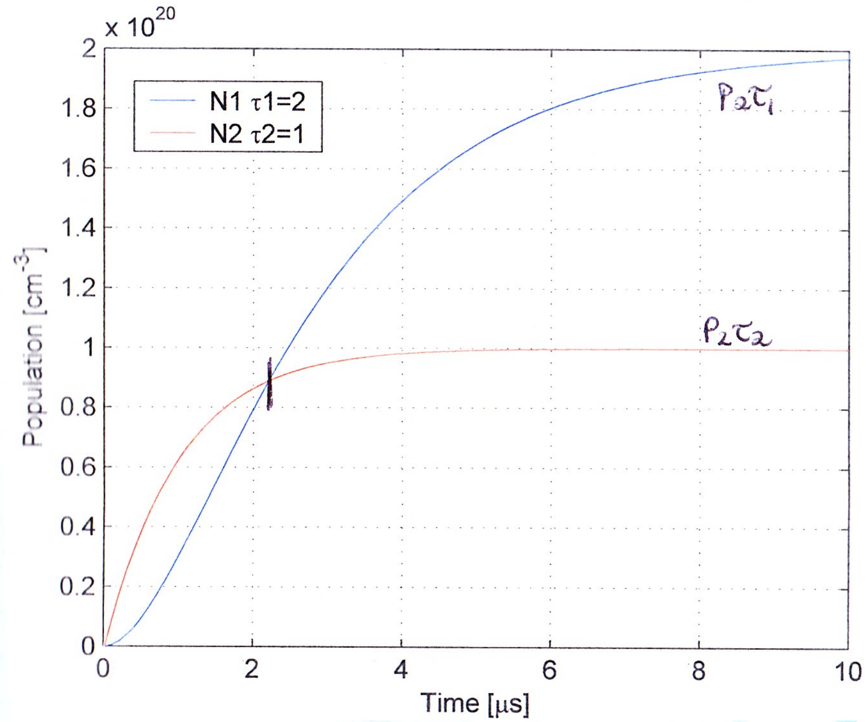
$t = 2\ln 3$

$t = 2.1972\ \mu s$

d) What are the steady-state populations?

$$\lim_{t\to\infty} N_1 = \lim_{t\to\infty} P_2\tau_1\left\{1 + \frac{\tau_1/\tau_2\, e^{-t/\tau_1}}{(1-\tau_1/\tau_2)} - \frac{e^{-t/\tau_2}}{(1-\tau_1/\tau_2)}\right\} = P_2\tau_1$$

$$\lim_{t\to\infty} N_2 = P_2\tau_2(1-e^{-t/\tau_2}) = P_2\tau_2$$

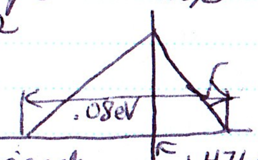



```
function Lasers_Ch7_Pr10
t=0:0.1:10.0;Tau1=2;Tau2=1;P2=10^20;
N2 = Tau2*P2*(1.0-exp(-t./Tau2));
N1 = Tau1*P2*(1.0+...
    (Tau1/Tau2)*(exp(-t./Tau1))/(1-Tau1/Tau2)...
    -(exp(-t./Tau2))/(1-Tau1/Tau2));
plot(t,N1,'b'),grid on;hold on;
plot(t,N2,'r'),grid on;hold on;
ylabel('Population [cm^-^3]');
xlabel('Time [\mus]');
Legend(['N1 \tau1=2';'N2 \tau2=1']);
```

7.11 The spontaneous emission profile of a certain laser can be approximated by the triangular shape shown below. If the spontaneous lifetim were 5nsec and the gain coef. were 10 cm^{-1}

a) The value of the line shape(s) at $h\nu_0$ = 1.476eV

$$\frac{\Delta\nu}{\nu} = \frac{\Delta E}{E} = \frac{0.08}{1.476} = 5.42\cdot10^{-2}$$



$\nu = 3.57\cdot10^{14}\,s^{-1}$

$\Rightarrow \Delta\nu = 19.4$ THz = base of triangle

$$\int g(\nu)d\nu = 1 = \tfrac{1}{2}\Delta\nu\, g(\nu_0) \Rightarrow g(\nu_0) = \frac{2}{\Delta\nu} = 1.03\,10^{-13}\,s$$

b) Inversion to obtain gain coefficient

$\lambda_0 = c/\nu = .84\,\mu m$ $\quad \gamma(\nu_0) = \frac{1}{\tau_s}\frac{\lambda_0^2}{8\pi} g(\nu_0)\Delta N$

$$\Delta N = \frac{\gamma(\nu_0)\,8\pi\tau_s}{\lambda_0^2\, g(\nu_0)} = \frac{(10cm^{-1})8\pi(5ns)}{(.84\mu m)^2(1.03\cdot10^{-13}s)} = \frac{1.7\cdot10^{15}}{cm^3}$$

7.12 Line shape of He-Ne $\lambda_0 = 6328$Å $(3s_2 - 2p_4)$ $A = 6.56\,10^{6}s^{-1}$

a) Value of $g(\nu_0)$?

$\int g(\nu_0) = 1 = K + 2(K)(3/4) + 2(K)(1/2) = 3.5K \quad K = 2/7 = g(\nu_0)$

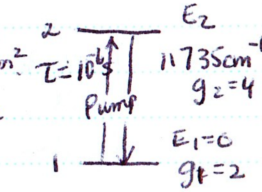
b) What is the stimulated emission cross-section?

$$\sigma(\nu) = \frac{A_{21}\lambda^2}{8\pi n^2} g(\nu) \qquad \sigma(\nu_0) = \frac{6.56\,10^6 s^{-1}(6328\cdot10^{-10})^2(\frac{2}{7})}{8\pi(1)^2}$$

$\sigma(\nu_0) = 2.99\cdot10^{-8} m^2 = 2.99\cdot10^{-12} cm^2$

c) $g(\nu)$ is the probability that a spontaneouly emitted photon is between ν and $\nu+d\nu$

7.13 Pump 2→1 and 1→2 $\tau_{spont} = 10^{-6}s$ $\sigma = 10^{-14}cm^2$



a) Rate equations $h\nu = (E_2 - E_1)$ degeneracy g_1, g_2

$$\frac{dN_2}{dt} = \frac{\sigma I_p}{h\nu}\left[\frac{g_2}{g_1}N_1 - N_2\right] - \frac{N_2}{\tau}$$

$N_1 + N_2 = N \quad \therefore \frac{dN_1}{dt} = -\frac{dN_2}{dt}$

b) What would be the population ratio N_2/N_1 if the intensity of the external wave were infinite?

$$\lim_{I_p\to\infty}\frac{dN_2}{dt} = -\frac{N_2}{\tau} \Rightarrow \frac{g_2}{g_1}N_1 - N_2 = 0 \Rightarrow \frac{N_2}{N_1} = \frac{g_2}{g_1}$$

c) I_p to get $N_2/N_1 = 1/2$ $\quad$ steady state $dN_2/dt = 0$

$$\frac{N_2}{\tau} = \frac{\sigma I_p}{h\nu}\left[\frac{g_2}{g_1}N_1 - N_2\right] \Rightarrow I_p = \frac{h\nu(N_2/N_1)}{\tau\sigma[g_2/g_1 - N_2/N_1]}$$

$$I_p = \frac{(11735-0)(1/2)cm^{-1}}{10^{-6}s\,[4/2 - 1/2]cm^2 10^{-6}} = 3.9\cdot10^{3}\frac{cm^{-1}s^{-1}}{cm^2}$$

$I_p = (3.9\cdot10^{3}cm^{-1}s^{-1}cm^{-2})(3\cdot10^{10}cms^{-1})(6.626\cdot10^{-34}Js)$

$I_p = 8\,W/cm^2$

d) If the ambient population temp = $KT = 208cm^{-1}$, $I_p = 0$ what is N_2/N_1

$$P(N_2) = \frac{g e^{-E_2/KT}}{\sum g_i e^{-E_i/KT}} = \frac{4e^{-11735/208}}{4e^{-11735/208} + 2e^0} \approx 10^{-25} \text{ practically zero.}$$

7.14 "Wrong" Blackbody derivation.

a) $dN_1/dt = -A_{21}N_2 + B_{12}N_1\rho(\nu) = 0 \Rightarrow \rho(\nu) = \frac{A_{21}}{B_{21}}\frac{N_2}{N_1} = \frac{A_{21}}{B_{21}}e^{-h\nu/KT}$

and $\rho(\nu) = \frac{A_{21}}{B_{12}}e^{-h\nu/KT}$ is $\lim_{h\nu/k\to\infty}\frac{h\nu}{e^{h\nu/KT}-1} \propto e^{-h\nu/KT}$

b) at optical frequencies $A_{21}/B_{12} = 8\pi h\nu^3/c^3$

c) Compare Rayleigh-Jeans, Wein, and Planck

$$\rho_W = \frac{8\pi h\nu^3}{c^3}e^{-h\nu/KT} \qquad \rho_{RJ} = \left(\frac{8\pi\nu^2}{c^3}\right)KT \qquad \rho_P = \frac{8\pi h\nu^3}{c^3}\left(\frac{1}{e^{h\nu/KT}-1}\right)$$

$$\frac{\rho_P}{\rho_{RJ}} = \frac{h\nu/KT}{e^{h\nu/KT}-1} \xrightarrow[\frac{h\nu}{KT}\ll1]{} 1 \qquad \frac{\rho_P}{\rho_W} = \frac{e^{h\nu/KT}}{e^{h\nu/KT}-1} \xrightarrow[\frac{h\nu}{KT}\gg1]{} 1$$