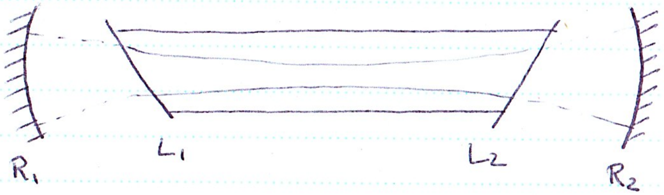
.10 Quartz windows oriented at Brewster's angle are commonly used for He:Ne lasers in the manner indicated in Figure 0.1. What is the angle from the axis of the cavity? (η[quartz] 1.43).

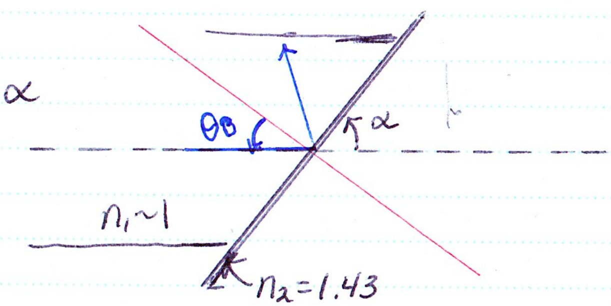


$\tan\theta_B = \frac{n_2}{n_1}$ Brewster angle

$\theta_B = \tan^{-1} 1.5 = .98\text{rad} = 56.3°$

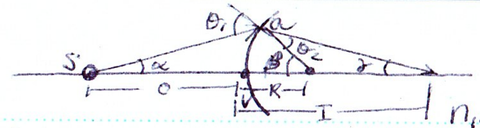
$\theta_B = \tan^{-1}\left(\frac{1}{1.5}\right) = .588\text{rad} = 33.69°$

$\theta_B = 90° - \alpha$



$\theta_B = \tan^{-1}\frac{n_2}{n_1} = \tan^{-1}\frac{1.43}{1} = 55.034°$

$\alpha = 90 - \theta_B \approx 35°$



$\theta_1 = \alpha + \beta \quad \beta = \theta_2 + \gamma \quad \alpha \approx \frac{aV}{O}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \beta \approx aV/R$

$n_1\theta_1 \sim n_2\theta_2 \quad \gamma \approx aV/I$

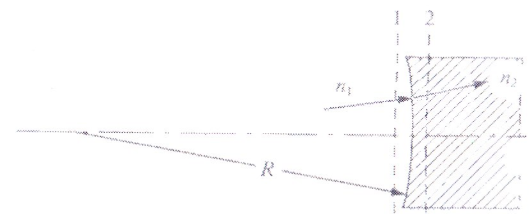
$n_1\alpha + n_2\gamma = (n_2 - n_1)\beta$

2.1 Derive the ray matrix for a ray entering a spherical dielectric interface.

$\rightarrow \frac{n_1}{O} + \frac{n_2}{I} = \frac{n_2 - n_1}{R}$

$\frac{aV}{O} + \frac{n_2 aV}{n_1 I} = \frac{\left(\frac{n_2}{n_1} - 1\right)aV}{R}$

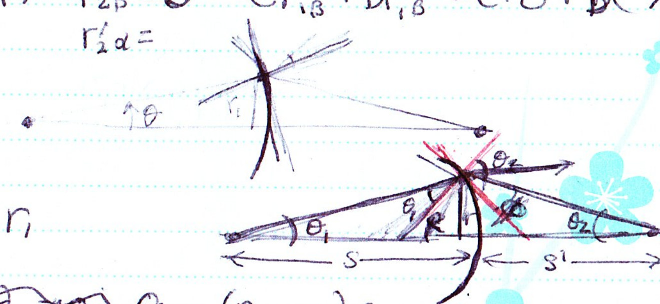
$\alpha + \frac{n_2}{n_1}\gamma = \frac{\beta\left(\frac{n_2}{n_1} - 1\right)}{R}$



$r'_{1\beta} = -r_2/R$

through #1: $r'_{2\beta} = 0 = Cr_{1\beta} + Dr'_{1\beta} = C \cdot O + D(-r_2/R)$

$r'_{2\alpha} =$



$r_2 = r_1$

~~$n_2 \sin\theta_2 = n_1 \sin\theta_1 - (n_2 - n_1)\frac{r_1}{R}$~~

$R\sin\theta = (R + s)\sin\theta_1 \quad R\sin\phi = (s' - r)\sin\theta_2$

$n_1 \sin\theta = n_2 \sin\phi$

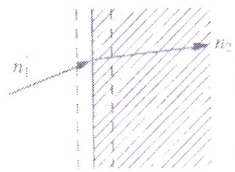
$\Rightarrow \frac{\sin\theta_1}{\sin\theta_2} = \frac{(s' - r)}{(s + r)}\frac{n_2}{n_1} \xrightarrow[\text{approx}]{\text{paraxial}} \frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$

$\sin\theta \sim \theta$

$r_2 = r_1 \quad \theta_1 = r'_1 = \frac{r_1}{s} \quad \theta_2 = \frac{-r_2}{s'} = r'_2$

$r'_2 = r'_1\left(\frac{n_1}{n_2}\right) - \left(1 - \frac{n_1}{n_2}\right)\frac{r_2}{R} \Rightarrow \begin{bmatrix} 1 & 0 \\ 1/R\left(\frac{n_1}{n_2} - 1\right) & \frac{n_1}{n_2} \end{bmatrix}$

2.2 Derive the ray matrix for the plane dielectric interface.



At the interface: $r_2 = r_1$

Snell's Law $\quad n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\sim n_1 \tan\theta_1 = \sim n_2 \tan\theta_2$

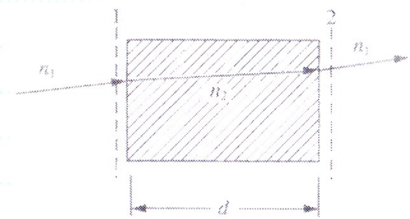
$n_1 r_1' = n_2 r_2'$

$r_2' = \left(\frac{n_1}{n_2}\right) r_1'$

$$\begin{bmatrix} r_2 \\ r_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{bmatrix} \begin{bmatrix} r_1 \\ r_1' \end{bmatrix}$$

$$\therefore T = \begin{bmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{bmatrix}$$

2.3 Derive the ray matrix for the plane dielectric slab of thickness d.



through interface #1: $r_2 = r_1$, $r_2' = (n_1/n_2) r_1'$ $\qquad T_1 = \begin{bmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{bmatrix}$

through slab ~~interface #2~~: $r_3 = r_2 + d r_2'$, $r_3' = r_2'$ $\qquad T_S = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$

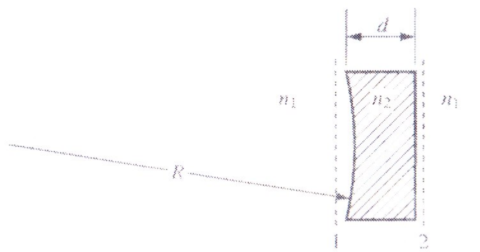
through interface #2: $r_4 = r_3$, $r_4' = (n_2/n_1) r_3'$ $\qquad T_2 = \begin{bmatrix} 1 & 0 \\ 0 & n_2/n_1 \end{bmatrix}$

$$T = T_2 T_S T_1 = \begin{bmatrix} 1 & 0 \\ 0 & n_2/n_1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & n_2/n_1 \end{bmatrix} \begin{bmatrix} 1 & (n_1/n_2)d \\ 0 & (n_1/n_2) \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & (n_1/n_2)d \\ 0 & 1 \end{bmatrix}$$

2.4 Combine the results of problems 1 and 2 to derive the ray matrix for the negative lens. $R >> d$



$$T_1 = \begin{bmatrix} 1 & 0 \\ (1-{}^{n_1}/_{n_2})\,{}^{1}/_{R} & {}^{n_1}/_{n_2} \end{bmatrix} \quad T_2 = \begin{bmatrix} 1 & 0 \\ 0 & {}^{n_2}/_{n_1} \end{bmatrix}$$

$$T_s = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

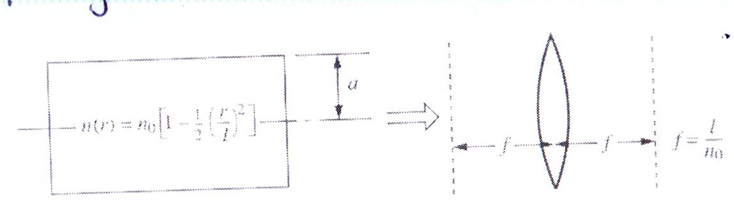
$$T = T_2 T_s T_1 = \begin{bmatrix} 1 & 0 \\ 0 & {}^{n_2}/_{n_1} \end{bmatrix}\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ (1-{}^{n_1}/_{n_2})\,{}^{1}/_{R} & {}^{n_1}/_{n_2} \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & {}^{n_2}/_{n_1} \end{bmatrix}\begin{bmatrix} 1+(1-{}^{n_1}/_{n_2})\,{}^{d}/_{R} & ({}^{n_1}/_{n_2})d \\ (1-{}^{n_1}/_{n_2})\,{}^{1}/_{R} & ({}^{n_1}/_{n_2}) \end{bmatrix}$$

$$T = \begin{bmatrix} 1+(1-{}^{n_1}/_{n_2})\,{}^{d}/_{R} & ({}^{n_1}/_{n_2})d \\ (1-{}^{n_1}/_{n_2})({}^{n_2}/_{n_1})\,{}^{1}/_{R} & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1+(1-{}^{n_1}/_{n_2})\,{}^{d}/_{R} & ({}^{n_1}/_{n_2})d \\ ({}^{n_2}/_{n_1}-1)\,{}^{1}/_{R} & 1 \end{bmatrix}$$

2.5 The GRIN lens shown in the diagram below consists of a graded index fiber with $n(r) = n_0[1-(r^2/2l^2)]$ of length d.



$$n(r) = n_0\left[1-\tfrac{1}{2}\left(\tfrac{r}{l}\right)^2\right]$$

interface #1: $T = \begin{bmatrix} 1 & 0 \\ 0 & {}^{1}/_{n(r)} \end{bmatrix}$ interface # 2: $T = \begin{bmatrix} 1 & 0 \\ 0 & n(r) \end{bmatrix}$

slab - GRIN lens

$\cancel{r_3 = r_2 + d(r)\, r_2'(r)}$ $\quad \begin{bmatrix} r_2 \\ r_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & {}^{1}/_{n(r)} \end{bmatrix}\begin{bmatrix} r_1 \\ r_1' \end{bmatrix}$

$\cancel{\Delta r_3 = r_2 + \int_0^d \frac{dl}{n(r)}}$

~~in slab~~

$\cancel{\frac{dr}{dz} = \frac{1}{n(r)}}$ for a continuous medium: 2.12.11

$$T = \begin{bmatrix} \cos(d/l) & l\sin(d/l) \\ -{}^{1}/_{l}\sin(d/l) & \cos(d/l) \end{bmatrix}$$ ray trace

matrix for $d = z$ through a continuous media for $n(r) = n_0(1 - r^2/2l^2)$ as $l \to \infty$ this becomes (2.2.2)

$$\lim_{l \to \infty} T = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Putting them together

$$T = T_{interface}\, T_{fiber}\, T_{interface}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & n_0 \end{bmatrix}\begin{bmatrix} \cos(d/a) & a\sin(d/a) \\ -1/a \sin(d/a) & \cos(d/a) \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1/n_0 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & n_0 \end{bmatrix}\begin{bmatrix} \cos(d/a) & a\sin(d/a)/n_0 \\ -\frac{1}{a}\sin(d/a) & 1/n_0 \cos(d/a) \end{bmatrix}$$

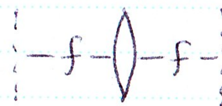
$$T = \begin{bmatrix} \cos(d/a) & a\sin(d/a)/n_0 \\ -n_0/a \sin(d/a) & \cos(d/a) \end{bmatrix}$$

$$\boxed{T = \begin{bmatrix} \cos(d/\ell) & \frac{\ell}{n_0}\sin(d/\ell) \\ -n_0/\ell \sin(d/\ell) & \cos(d/\ell) \end{bmatrix}} = \begin{bmatrix} \cos d/\ell & \ell/n_0 \sin d/\ell \\ -n_0/\ell \sin d/\ell & \cos d/\ell \end{bmatrix}$$

b) Evaluate for $d = \pi\ell/2$.

$$T = \begin{bmatrix} \cos(\pi/2) & \frac{\ell}{n_0}\sin(\frac{\pi\ell}{2\ell}) \\ -n_0/\ell \sin(\pi/2) & \cos(\pi\ell/2\ell) \end{bmatrix} = \begin{bmatrix} 0 & \frac{\ell}{n_0} \\ -n_0/\ell & 0 \end{bmatrix}$$

c) Show that this lens is equivalent to the system shown at the right.



$$f = \frac{\ell}{n_0}$$

$$T = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & d \\ -1/f & 1-d/f \end{bmatrix}$$

$$T = \begin{bmatrix} 1-d/f & d+d-d^2/f \\ -1/f & 1-d/f \end{bmatrix} = \begin{bmatrix} 1-d/f & 2d-d^2/f \\ -1/f & 1-d/f \end{bmatrix}$$

$d \to f \qquad T = \begin{bmatrix} 0 & f \\ -1/f & 0 \end{bmatrix}$ same as $f \to \frac{\ell}{n_0}$ $\begin{bmatrix} 0 & \ell/n_0 \\ -n_0/\ell & 0 \end{bmatrix}$

d) If $n_0 = 1.53 \quad n(a) = 1.525$
$a = 2mm$ find f.

$$n = n_0\left[1 - \frac{r^2}{2\ell^2}\right]$$

$\ell = 0 \quad n = n_0 = 1.53$

$\ell = a \quad n = n_0[1 - a^2/2\ell^2] = 1.525$

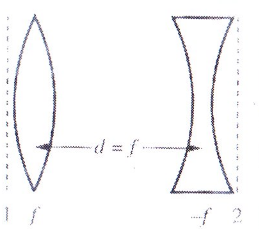
$$1 - a^2/2\ell^2 = \frac{1.525}{n_0}$$

$$a^2/2\ell^2 = (1 - \frac{1.525}{n_0}) \Rightarrow \ell^2/n_0^2 = \frac{a^2}{2n_0^2}(1 - \frac{1.525}{n_0})^{-1}$$

$$f^2 = \frac{\ell^2}{n_0^2} = \frac{a^2}{2n_0^2}\left(1 - \frac{n(a)}{n_0}\right)^{-1} = \frac{(2mm)^2}{2(1.53)^2}\left(1 - \frac{1.525}{1.53}\right)^{-1}$$

$$f = 16.169 \text{ mm}$$

.6 Find the ABCD matrix for the lens combination.



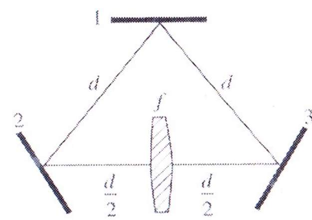
$T_1 = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$ $T_2 = \begin{bmatrix} 1 & 0 \\ 1/f & 1 \end{bmatrix}$ $T_d = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & f \\ 0 & 1 \end{bmatrix}$

$$T = T_2 T_d T_1 = \begin{bmatrix} 1 & 0 \\ 1/f & 1 \end{bmatrix}\begin{bmatrix} 1 & f \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$

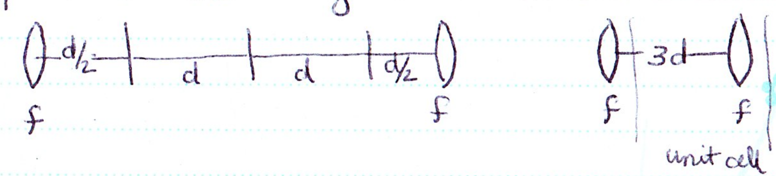
$$T = \begin{bmatrix} 1 & 0 \\ 1/f & 1 \end{bmatrix}\begin{bmatrix} 1-1 & f \\ -1/f & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/f & 1 \end{bmatrix}\begin{bmatrix} 0 & f \\ -1/f & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & f \\ -1/f & f/f+1 \end{bmatrix} = \begin{bmatrix} 0 & f \\ -1/f & 2 \end{bmatrix}$$

2.7 Consider the ring laser cavity.



a) Equivalent-lens waveguide   unit cell



b) Transmission matrix

$$T_d = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}\begin{bmatrix} 1 & 3d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3d \\ -1/f & 1-3d/f \end{bmatrix}$$

c) What are the values of d/f that make this a stable cavity?

Stability Condition: $-1 \le \frac{A+D}{2} \le +1$

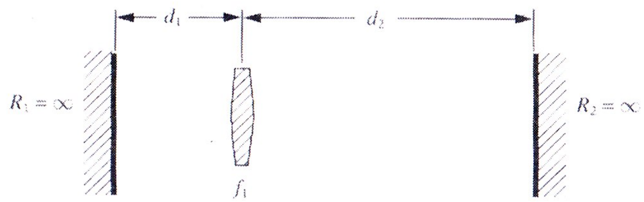
$$-1 \le \left(\frac{1+1-3d/f}{2}\right) \le 1$$

$$-1 \le 1 - 3d/2f \le 1$$

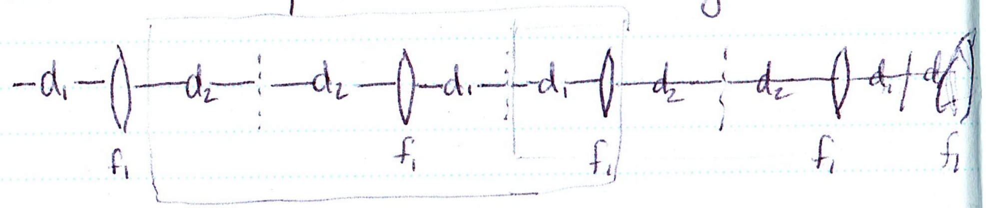
$$-2 \le -3d/2f \le 0$$

$$\boxed{0 \le d/f \le 4/3}$$

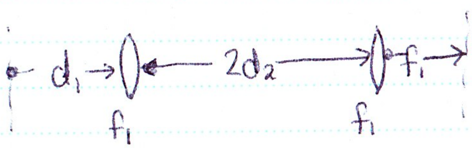
8 Consider the cavity:



) Construct an equivalent lens waveguide.



) Indicate a unit cell starting at a flat mirror, R_1:



) Find the ray matrix for the unit cell of (b):

$T_1 = \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix}$ $T_2 = \begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix}$ $T_3 = \begin{bmatrix} 1 & 2d_2 \\ 0 & 1 \end{bmatrix}$ $T_4 = \begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix}$ $T_5 = \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix}$

$$T = T_5 T_4 T_3 T_2 T_1 = \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2d_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix}\begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix}$$

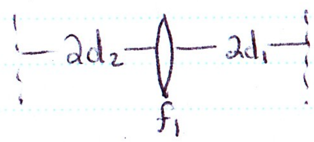
$$= \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2d_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & d_1 \\ -1/f_1 & 1-d_1/f_1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix}\begin{bmatrix} 1 - 2d_2/f_1 & d_1 + 2d_2(1-d_1/f_1) \\ -1/f_1 & 1-d_1/f_1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 - 2d_2/f_1 & d_1 + 2d_2(1-d_1/f_1) \\ -1/f_1(1-2d_2/f_1) - 1/f_1 & -1/f_1(d_1 + 2d_2(1-d_1/f_1)) + 1 - d_1/f_1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 - 2d_2/f_1 - d_1/f_1(2 - 2d_2/f_1) & d_1 + 2d_2(1-d_1/f_1) - d_1/f_1(d_1 + 2d_2(1-d_1/f_1)) + d_1 - d_1^2/f_1 \\ -1/f_1(1 - 2d_2/f_1) - 1/f_1 & -1/f_1(d_1 + 2d_2(1-d_1/f_1)) + 1 - d_1/f_1 \end{bmatrix}$$

or



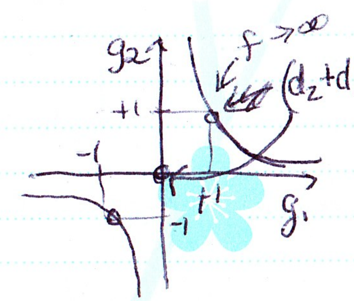
$$T = \begin{bmatrix} 1 & 2d_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2d_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2d_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2d_1 \\ -1/f_1 & -2d_1/f_1 + 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 + (-2d_2/f_1) & 2d_1 - 4d_1d_2/f_1 + 2d_2 \\ -1/f_1 & -2d_1/f_1 + 1 \end{bmatrix}$$

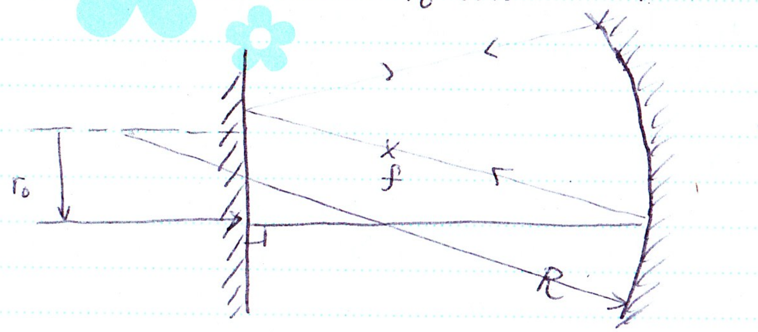
$$0 \le \frac{A+D+2}{4} \le 1 \text{ stable} \Rightarrow 0 \le \frac{2 - 2d_2/f_1 - 2d_1/f_1 + 2}{4} \le 1$$

$$\Rightarrow 0 \le 1 - \frac{1}{2f_1}(d_2 + d_1) \le 1$$

$$g_1 g_2 = 1 - \frac{1}{2f_1}(d_2 + d_1)$$



.9 Sketch the ray pattern shown in Fig. 2.10 if $d/R = 1/2$. Assume the initial ray is a distance r_0 below the axis.



a) What is the maximum excursion of the path from the axis? $d/R = 1/2 \Rightarrow d = 2R = f$

$$g_1 = 1 - \frac{d}{R_1} = 1 \qquad g_2 = 1 - d/R_2 = 1 - 1/2 = 1/2$$

$$g_1 g_2 = (1)(\tfrac{1}{2}) = \tfrac{1}{2} < 1 \quad \therefore \text{ stable}$$

$$T = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2/R_2 & 1 \end{bmatrix}\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

$$\cancel{T = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & d \\ -2/R_2 & d-2/R_2 \end{bmatrix} = \begin{bmatrix} 1-2d/R_2 & d-d^2-2d/R_2 \\ -2/R_2 & d-2/R_2 \end{bmatrix}}$$

$$\cancel{T = \begin{bmatrix} 0 & d-d^2 \end{bmatrix}} \quad T = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & d \\ -1/f & 1-d/f \end{bmatrix}$$

$$T = \begin{bmatrix} 1-d/f & d+d-d^2/f \\ -1/f & 1-d/f \end{bmatrix} = \begin{bmatrix} 1-d/f & 2d-d^2/f \\ -1/f & 1-d/f \end{bmatrix}$$

$d = f$

$$T = \begin{bmatrix} 1-f/f & 2f-f^2/f \\ -1/f & 1-f/f \end{bmatrix} = \begin{bmatrix} 0 & f \\ -1/f & 0 \end{bmatrix} = \begin{bmatrix} 0 & d \\ -1/d & 0 \end{bmatrix}$$

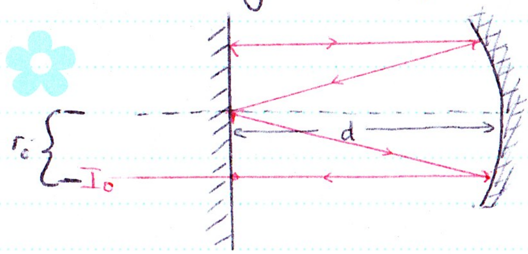
$$\cos\theta = \left(\frac{A+D}{2}\right) = 0 \Rightarrow \theta = \pi/2$$

$$\alpha = \tan^{-1}\left\{\frac{a\left[1-\left(\frac{A+D}{2}\right)^2\right]^{1/2}}{a\left(\frac{A-D}{2}\right) + Bm}\right\} = \tan^{-1}\left\{\frac{a}{Bm}\right\}$$

$a = -r_0 \quad r_0' = m = -r_0/d$

$$\alpha = \tan^{-1}\left\{\frac{-r_0}{d\left[-r_0/d\right]}\right\} = \tan^{-1}(1) = \pi/4$$

Figure 2.10 shows 3 round trips to obtain repetition, how many are required here?



$\frac{d}{R} = \frac{1}{2}$

$R = 2f$

$d = f$

4 round trips are needed.

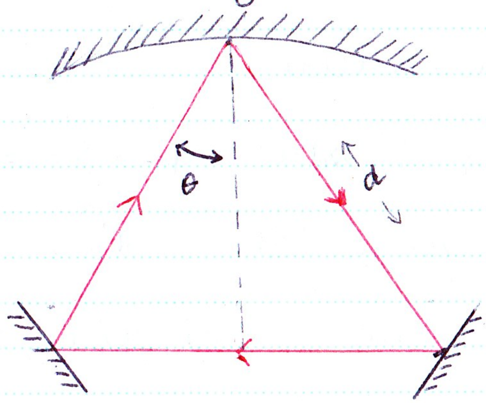
① round trip to mirror - parallel in reflects of mirror to intersect focal point - which is "d" the distance between the two mirrors

② Roundtrip #2 equal angle from axis reflects off mirror to parallel ray back to flat mirror

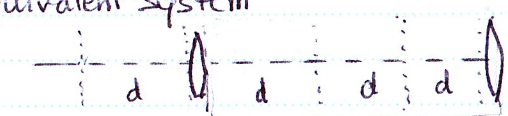
③ Round trip #3 the incident light is parallel - it reflects off the flat mirror parallel then off the spherical mirror at angle to intersect at focal point (on optic axis)

④ Round trip #4 is the reflection of Round trip #2 about the optical axis Completes the path of the photon to its original point of entry

2.11 In the analysis of the cavity of Fig. 2-13 one must keep track of displacements from the chief ray in the plane of and perpendicular to the paper. If the chief ray forms an equilateral triangle with sides d and the spherical mirror has a radius of curvature R, what is the limit of d/R to ensure stability?



equivalent system



$T_R = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}$ $\quad T_2 = \begin{bmatrix} 1 & 3d \\ 0 & 1 \end{bmatrix}$

$T = T_R T_2 = \begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}\begin{bmatrix} 1 & 3d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3d \\ -2/R & -6d/R+1 \end{bmatrix}$

$T = \begin{bmatrix} 1 & 3d \\ -1/f & 1-3d/f \end{bmatrix} = \begin{bmatrix} 1 & 3d \\ -2/R & 1-6d/R \end{bmatrix}$

$f_x = f\cos\theta \quad f_y = \frac{f}{\cos\theta}$

θ for equilateral triangle $= 30°$

$\cos\theta = \frac{\sqrt{3}}{2}$

$f_x = \frac{\sqrt{3}}{2} f \qquad f_y = \frac{2}{\sqrt{3}} f$

Stability Condition: $-1 \le \left(\frac{A+D}{2}\right) \le +1$

$A = 1 \quad D_x = 1 - 3d/f_x \quad D_y = 1 - 3d/f_y \quad R = 2f$

in the xy plane $f_x = \sqrt{3}/2 f = (\sqrt{3}/2)(R/2) = \sqrt{3}/4 R$

$A + D_x = 1 + 1 - 3d/f_x = 2 - 3d/(\sqrt{3}R/4) = 2 - \frac{12}{\sqrt{3}}\frac{d}{R}$

$-1 \le \frac{2 - \frac{12}{\sqrt{3}}(d/R)}{2} \le +1$

$-1 \le 1 - \frac{6}{\sqrt{3}}\left(\frac{d}{R}\right) \le +1$

$-2 \le -\frac{6}{\sqrt{3}}\frac{d}{R} \qquad -\frac{6}{\sqrt{3}}\left(\frac{d}{R}\right) \le 0$

$1 \ge \frac{3}{\sqrt{3}}\frac{d}{R}$

$\boxed{\frac{\sqrt{3}}{3} \ge \frac{d}{R}} \qquad -\frac{d}{R} \le 0$

xy plane

in the xz plane

$A + D_y = 1 + 1 - 3d/f_y = 2 - \frac{3d\sqrt{3}}{2f} = 2 - \frac{3\sqrt{3}d}{R}$

$-1 \le \frac{A + D_y}{2} \le +1$

$-1 \le \frac{2 - \frac{3\sqrt{3}d}{2R}}{2} \le +1$

$-1 \le 1 - \frac{3}{2}\sqrt{3}\left(\frac{d}{R}\right) \le 1$

$-2 \le -\frac{3}{2}\sqrt{3}\left(\frac{d}{R}\right) \qquad -\frac{d}{R} \le 0$

$2 \ge \frac{3}{2}\sqrt{3}\left(\frac{d}{R}\right)$

$\frac{d}{R} \le \frac{4\sqrt{3}}{3}$ in the xz plane.

2.14 Typical parameters for a glass fiber are radius $= 20\mu m = a$
$n_0 = 1.5 \quad \Delta n = 8 \cdot 10^{-3}$
How many times would a ray cross the axis of a fiber $\underbrace{1 \text{ km long}}_{z_1}$?

$r(z) = r_1 \cos\left(\frac{z}{\ell}\right) + r_1' \,\ell \sin\left(\frac{z}{\ell}\right)$

$r(z)$ is a maximum when $r_1' = 0$ and
$r_1 = r_0$ or $-r_0$ $\quad \frac{z}{\ell} = \pi$ for each crossing

$\Delta z = \pi \ell = \pi a \sqrt{\frac{n_0}{2\Delta n}}$ = distance between crossings

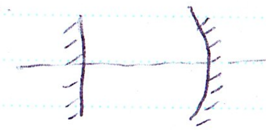
number of crossings = $\frac{\text{total length}}{\text{distance between crossings}} + 1$

$\# \text{crossings} = \frac{\cancel{1000cm}\, z_1 \sqrt{2\Delta n}}{\pi a \sqrt{n_0}}$

$\# \text{crossings} = \frac{1000m \sqrt{2}\sqrt{8 \cdot 10^{-3}}}{\pi (20 \cdot 10^{-6} m)\sqrt{1.5}} + 1 = 59{,}366$

2.15 Is it possible for $AD - BC \neq 1$ for a unit cell of an optical cavity?
$\begin{bmatrix} r_2 \\ \theta_2 \end{bmatrix} = \lambda \begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix} \Rightarrow \begin{vmatrix} A-\lambda & B \\ C & D-\lambda \end{vmatrix} = 0 \Rightarrow$
$(A-\lambda)(D-\lambda) - BC = 0$
$AD - \cancel{AD} - \lambda(A+D) + \lambda^2 - BC = 0 \Rightarrow \lambda^2 - (A+D)\lambda + (AD - BC) = 0$
$\lambda = \frac{(A+D) \pm \sqrt{(A+D)^2 - 4(AD - BC)}}{2}$

6.1 $d = \frac{3}{4} R_2 \quad \Gamma_1^2 = .99 \quad \Gamma_2^2 = .97$
a) expression for resonant frequencies of TEM_{00} modes of cavity



$R_1 \quad R_2$