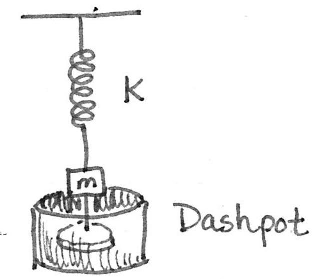
3.2)



$m = 100g$ $k = 10^4$ dyne cm^{-1}

$v_0 = 0$

$x - x_0 = 3\,cm$

at $t = 10s$ $A = \frac{A_0}{2}$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$$

(a) find damping parameter:

$$x_{en} = A_0 e^{-\beta t}$$
$$(0.5)A_0 = A_0 e^{-\beta(10s)}$$
$$-\beta(10s) = \ln(0.5)$$
$$\beta = -\tfrac{1}{10s}\ln(0.5)$$
$$\beta = 0.0693\,s^{-1}$$

(b) find the frequencies:

$$\omega_0 = (km^{-1})^{1/2} = (10^4 \text{ dyne } cm^{-1}\, 100g^{-1})^{1/2} = 1{,}000(\text{dyne } cm^{-1} g^{-1})^{1/2} = 1{,}000s$$

$$\nu_0 = \frac{\omega_0}{2\pi} = \frac{500}{\pi}(\text{dyne } cm^{-1} g^{-1})^{1/2} = \frac{500}{\pi s}$$

$$\omega_1 = (\omega_0^2 - \beta^2)^{1/2} = \left((1000)^2 - (0.0693)^2\right)^{1/2} s^{-1} \cong 1{,}000 s^{-1}$$

$$\nu_1 \sim \frac{500}{\pi s}$$

$$\nu_1 \sim \nu_0$$

c.) find decrement of motion:

$$\tau_1 = \frac{2\pi}{\omega_1} = \frac{2\pi\, s}{1000} \qquad \beta = 0.0693 s^{-1}$$

$$\exp(\tau_1\beta) = e^{\left[\frac{2\pi s}{1000}\right]\left[0.0693 s^{-1}\right]} = 1.0004$$

3.4.) <u>time averages:</u>

potential $U = \frac{1}{2}kA^2\sin^2(\omega_0 t - \delta)$

$$\langle U \rangle_t = \frac{\int_t^{t+2\pi} U\,dt}{2\pi} = \frac{\int_t^{t+2\pi} \frac{1}{2}kA^2\sin^2(\omega_0 t-\delta)}{2\pi} = \left(\frac{\frac{1}{4}kA^2}{2\pi}\right)$$

kinetic $T = \frac{1}{2}m\dot{x}^2$

$$\langle T \rangle_t = \frac{\int_t^{t+2\pi} T\,dt}{2\pi} = \frac{\int_t^{t+2\pi} \frac{1}{2}kA^2\cos^2(\omega_0 t-\delta)\,dt}{2\pi} = \left(\frac{\frac{1}{4}kA^2}{2\pi}\right)$$

$$\therefore \langle U \rangle_t = \langle T \rangle_t$$

<u>space averages:</u>

potential $U = E - \frac{1}{2}m\dot{x}^2 = \frac{1}{2}kx^2$

$$\langle U \rangle_x = \frac{\int_x^{x+h} \frac{1}{2}kx^2\,dx}{h} = \frac{k}{2h}\left(\frac{x^3}{3}\right)\Bigg|_x^{x+h} = \frac{kh^3}{6h} = \frac{kh^2}{6}$$

kinetic $K = E - \frac{1}{2}kx^2$

$$\langle T \rangle_x = \frac{\int_x^{x+h} E - \frac{1}{2}kx^2\,dx}{h} = \frac{Eh}{h} - \frac{kh^2}{6} = E - \frac{kh^2}{6}$$

This seems reasonable because the sum of the space averaged kinetic and potential energy is a constant.

3-6

horizontal frictionless surface

① $m_1 a_1 = m_1 \ddot{x}_1 = +kx$

② $m_2 a_2 = m_2 \ddot{x}_2 = -kx$

① $m_2 m_1 \ddot{x}_1 = m_2 kx$ ② $m_1 m_2 \ddot{x}_2 = -m_1 kx$

② - ① =

$$-m_2 m_1 \ddot{x}_1 + m_1 m_2 \ddot{x}_2 = -m_1 kx - m_2 kx$$

$$m_1 m_2 \frac{d^2}{dt^2}(x_2 - x_1) = -(m_1 + m_2) kx$$

$$m_1 m_2 \frac{d^2 x}{dt^2} = -(m_1 + m_2) kx$$

$$\ddot{x} + \left[\frac{k}{\left(\frac{m_1 m_2}{m_1 + m_2}\right)}\right] x = 0$$

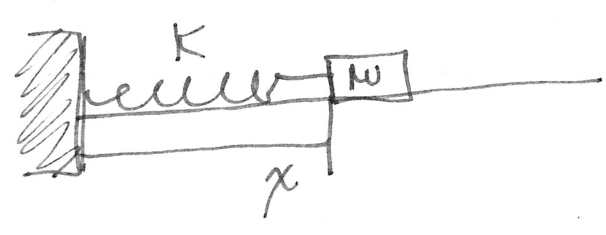
$$x = c_1 e^{i\omega t} + c_2 e^{-i\omega t} = c_1 \cos\omega t + c_2 \sin\omega t$$

where $\omega = \sqrt{\frac{k}{\left(\frac{m_1 m_2}{m_1 + m_2}\right)}}$ $\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{\frac{m_1 m_2}{(m_1 + m_2)}}}$

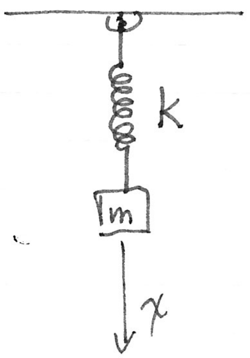
We can define reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$

so that $\omega = \sqrt{\frac{k}{\mu}}$

equivalent system:



3-9.)



$$\ddot{x} + \omega_0^2 x = \frac{F}{m} \quad \text{for } t \le t_0$$

$$\ddot{x} + \omega_0^2 x = 0 \quad \text{for } t \ge t_0$$

$x_H = A\cos(\omega_0 t)$ set $\phi = 0$

method of undetermined coefficients:

$x_p = \cancel{F}\ K_0$

substitute

$(D^2 + \omega_0^2)\, x_p = \frac{F}{m}$

$$\omega_0^2 K_0 = \frac{F}{m}$$

$$K = \frac{F}{\omega_0^2 m} = \frac{F}{k}$$

thus for $t \le t_0$

$$x = A\cos\omega_0 t + \frac{F}{k}$$

at $t = 0$ $x = x_0$

$$x_0 = A\cos 0 + \frac{F}{k} \Rightarrow A = x(0) - \frac{F}{k}$$

~~ytt~~ $x(t_0) = \left[x_0 - \frac{F}{k}\right]\cos\omega_0 t_0 + \frac{F}{k}$

and for $t \ge t_0$

$$(x - x_0) = A_1 \cos\omega_0(t - t_0)$$

$$0 = x(t_0) = A_1\cos\omega_0(0) = \left[x_0 - \frac{F}{k}\right]\cos\omega_0 t_0 + \frac{F}{k}$$

$$x - x_0 = \left(\left[\cancel{x_0}^{\,0} - \frac{F}{k}\right]\cos\omega_0 t_0 + \frac{F}{k}\right)\cos\omega_0(t - t_0)$$

$$x - x_0 = \frac{F}{x}\left[\cos\omega_0(t - t_0) - \cos\omega_0 t_0\right]$$

3.10.) $A = A_0 e^{-\beta t}$

at $t = nT$ $\quad A = \frac{A_0}{e}$

$A_0 e^{-1} = A_0 e^{-\beta T n}$

$\beta T n = 1$

$$\beta = \frac{1}{nT} = \frac{\omega_1}{n 2\pi}$$

$$\omega_0^2 = \omega_1^2 + \beta^2$$

$$\omega_0^2 = \omega_1^2 + \omega_1^2 (n 2\pi)^{-2}$$

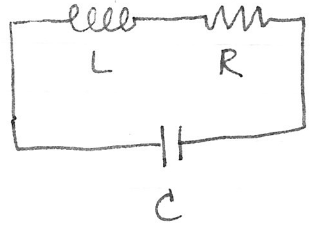
$$\omega_0^2 = \omega_1^2 (1 + (n 2\pi)^{-2})$$

$$\omega_1 = \omega_0 (1 + (n \cancel{2}\pi)^{-2})^{-1/2}$$

by binomial expansion:

$$\omega_1 \sim \omega_0 [1 - (8\pi^2 n^2)^{-1}]$$

3.17) $L = 0.1$ Henry
$C = 1.0 \cdot 10^{-3}$ Farad
$R = 100\ \Omega$



find frequency of oscillation:

$$L\ddot{q} + R\dot{q} + \frac{1}{C}(q) = 0$$

$$\ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{LC}q = 0$$

analogous to $\quad \ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$

$$\omega_0 = \left(\frac{1}{LC}\right)^{1/2} = \left(\frac{1}{(0.1\,H)(1 \cdot 10^{-6}F)}\right)^{1/2} = 3{,}162\ s^{-1}$$

$$\beta = \frac{R}{2L} = \frac{100\ \Omega}{2(0.1\ H)} = 500\ s^{-1}$$

$$\omega_1 = (\omega_0^2 - \beta^2)^{1/2} = (3{,}162^2 - 500^2)^{1/2}\ s^{-1} = 3122\ s^{-1}$$

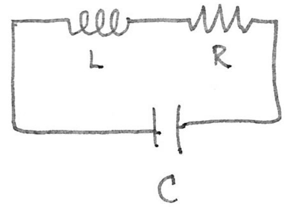
$$\nu_1 = \frac{\omega_1}{2\pi} = 467\ s^{-1}$$

3.19) $L = 0.01\ H$

$R = 100\Omega$

$\omega_1 = 1\cdot 10^3 s^{-1}$

$\beta = \frac{R}{2L} = 5000 s^{-1}$



$\ddot{q} + 2\beta\dot{q} + \omega_0^2 q = 0$

find C:

$\omega_1^2 = \omega_0^2 - \beta^2$

$LC = (\omega_1^2 + \beta^2)^{-1}$

$C = \frac{1}{L}(\omega_1^2 + \beta^2)^{-1} = (0.01)^{-1}\left[1\cdot 10^6 + \left(\frac{100}{2(0.1)}\right)^2\right]^{-1} = 3.85\cdot 10^{-6} F$

differential equation solution

$q = e^{-\beta t}\left[A_1 e^{+\omega_1 t} + A_2 e^{-\omega_1 t}\right]$

$i = \dot{q} = -\beta e^{-\beta t}\left[A_1 e^{\omega_1 t} + A_2 e^{-\omega_1 t}\right] + e^{-\beta t}\left[A_1 \omega_1 e^{\omega_1 t} + A_2(-\omega_1)e^{-\omega_1 t}\right]$

initial values: $q(0) = CV(0) = 3.85\cdot 10^{-5} C$

$V(0) = 10V$ $\quad i(0) = 0$

$3.85\cdot 10^{-5} = A_1 + A_2$ $\qquad 0 = -5000[A_1 + A_2] + 1000[A_1 - A_2]$

$A_1 = -\frac{3}{2}A_2$

$3.86\cdot 10^{-5} = -\frac{3}{2}A_2 + A_2$

$A_2 = 7.7\cdot 10^{-5} \Rightarrow A_1 = -\frac{3}{2}A_2 = -11.55\cdot 10^{-5}$

now find current at $t = 0.2\cdot 10^{-3} s$

$i = -5000\left[e^{(-5000)2\cdot 10^{-4}}\right]\left[-11.55\cdot 10^{-5} e^{.2} + 7.7\cdot 10^{-5} e^{-0.2}\right]$

$+ e^{(-5000)2\cdot 10^{-4}}[1000]\left[-11.55\cdot 10^{-5} e^{.2} - 7.7\cdot 10^{-5} e^{-0.2}\right]$

$i = -5000[0.368][-7.8\cdot 10^{-5}] + [1{,}000][0.368][-0.0002]$

$i = 0.1435 + (-0.075)$

$i = 0.0684\ A$