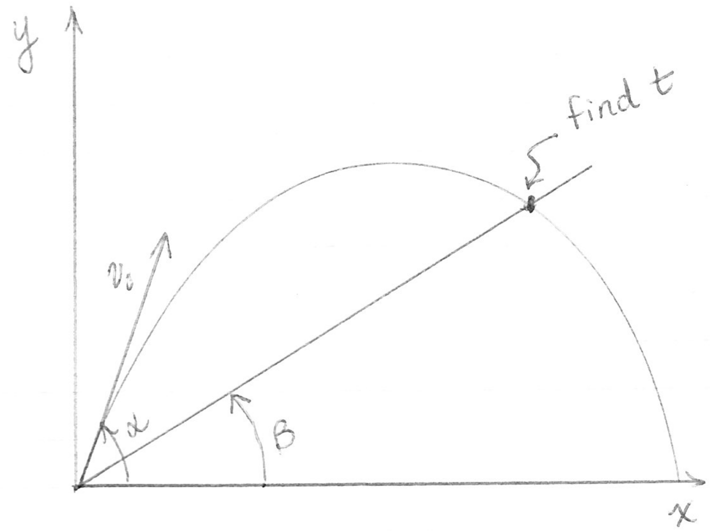
Theoretical Mechanics M. A. Webster
Ch. 2 Newtonian Mechanics

$\frac{80}{50}$

2.3) particle projectile v_0 - at origin at angle α
Calculate time to cross line passing through origin at angle β $\beta < \alpha$



$x = v_{0x} t = (v_0 \cos\alpha) t$

$y = v_{0y} t - \frac{1}{2} g t^2$

$y = (v_0 \sin\alpha) t - \frac{1}{2} g t^2$

required that:

$$\frac{y}{x} = \tan\beta$$

substitute:

$$(v_0 \sin\alpha) t - \tfrac{1}{2} g t^2 = (v_0 \cos\alpha) t (\tan\beta)$$

solve for t:

$$t \left[(v_0 \sin\alpha) - v_0 (\cos\alpha)(\tan\beta) - \tfrac{1}{2} g t \right] = 0$$

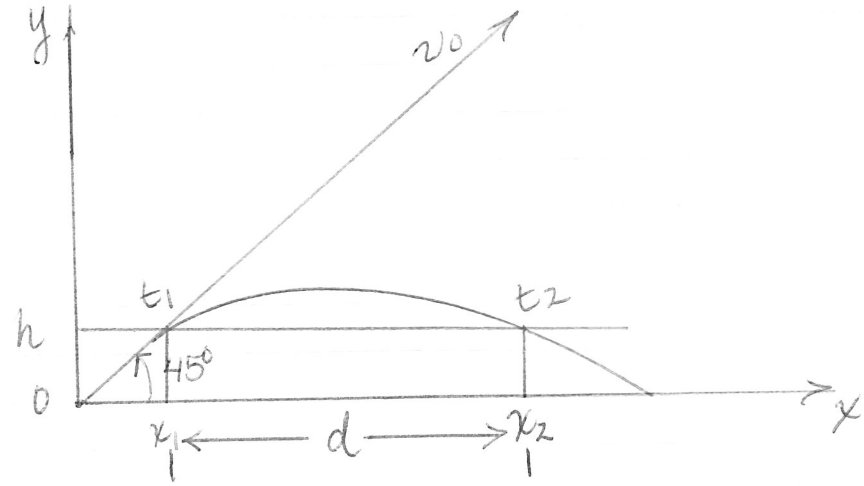
$\cancel{t = 0}$ throw out this solution

$$t = \frac{2 (v_0 \sin\alpha - v_0 \cos\alpha \tan\beta)}{g}$$

$$t = \frac{2 v_0 (\sin\alpha \cos\beta - \cos\alpha \sin\beta)}{g \cos\beta}$$

$$\boxed{t = \frac{2 v_0 \sin(\alpha - \beta)}{g \cos\beta}}$$

2.4.) projectile : v_0 at maximum range ∴ 45° with horizontal
projectile passes through to points of $y=h$ find $\Delta x = d$.



$$y = (v_0 \sin 45°)t - \tfrac{1}{2}gt^2 = \frac{v_0\sqrt{2}}{2}t - \frac{g}{2}t^2$$

$$x = (v_0 \cos 45°)t = \frac{v_0\sqrt{2}}{2}t$$

requirement: $y = h$ substitute and solve for t

$$h = \frac{v_0\sqrt{2}}{2}t - \frac{gt^2}{2}$$

$$\frac{gt^2}{2} - \frac{v_0\sqrt{2}}{2}t + h = 0 \quad \therefore \quad t = \frac{\frac{v_0\sqrt{2}}{2} \pm \sqrt{\left(\frac{v_0\sqrt{2}}{2}\right)^2 - 4\left(\frac{g}{2}\right)h}}{2\left(\frac{g}{2}\right)}$$

thus at x_1 : $t_1 = \frac{v_0\sqrt{2}}{2g} - \frac{\sqrt{\frac{v_0^2}{2} - 2gh}}{g}$ and at x_2 : $t_2 = \frac{v_0\sqrt{2}}{2g} + \frac{\sqrt{\frac{v_0^2}{2} - 2gh}}{g}$

and $\Delta t = t_2 - t_1 = \frac{2}{g}\sqrt{\frac{v_0^2}{2} - 2gh}$

to find separation of these points.

$$d = \Delta x = \frac{v_0\sqrt{2}}{2}\Delta t = \frac{v_0\sqrt{2}}{\cancel{2}}\left(\frac{\cancel{2}}{g}\sqrt{\frac{v_0^2}{2} - 2gh}\right) = \boxed{\frac{v_0}{g}\sqrt{v_0^2 - 4gh} = d}$$

2.9.) particle moves in medium $F = mk(v^3 + a^2 v)$ where a, k are constants find dmax and Show $v = 0$ at $t \Rightarrow \infty$.

given $F = mk(v^3 + a^2 v)$

$$m\frac{dv}{dt} = mk(v^3 + a^2 v)$$

$$\frac{dv}{dt} = k(v^3 + a^2 v)$$

find $v = f(x)$:

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dt}$$

$$\therefore \frac{dv}{dx} = \frac{1}{v}\frac{dv}{dt}$$

Subsititute:

$$\frac{dv}{dx} = \frac{1}{v}(kv^3 + a^2 v)$$

$$\frac{dv}{dx} = k(v^2 + a^2)$$

$$\int \frac{dv}{v^2 + a^2} = \int k\,dx$$

$$kx = \frac{1}{a}\tan^{-1}\frac{v}{a} + \cancel{C}^{\,0}$$

$$\cancel{v = \frac{dx}{dt}} =$$

$$\tan^{-1}\frac{v}{a} = akx \quad \text{for maximum } x \text{ let } v \to \infty$$

$$\pi/2 = \tan^{-1}\infty = akx$$

$$\boxed{x = \frac{\pi}{ak2}}$$

now show that $v=0$ at $t \to \infty$

$$\frac{dv}{dt} = -k(v^3 + a^2 v) \qquad \text{resisting medium}$$

$$-\int \frac{dv}{(v^3+a^2v)} = \int k\,dt$$

$$-\frac{1}{2a^2} \ln\left(\frac{v^2}{v^2+a^2}\right) = \cancel{2kt}\ kt$$

$$\ln\left(\frac{v^2}{v^2+a^2}\right) = -2akt$$

$$\frac{v^2}{v^2+a^2} = e^{-2akt}$$

$$\frac{v^2+a^2}{v^2} = e^{+2akt}$$

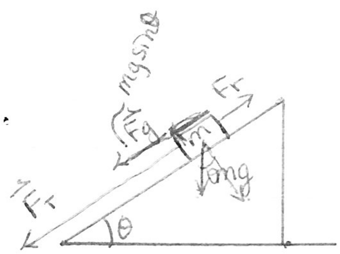
$$\frac{a^2}{v^2} = (e^{+2a^2kt} - 1)$$

$$v^2 = \frac{a^2}{(e^{+2a^2kt} - 1)}$$

$$v = \left(\frac{a}{e^{+2a^2kt} - 1}\right)^{1/2}$$

$$v\Big|_{t=\infty} = \lim_{t\to\infty}\left(\frac{a}{e^{2a^2kt} - 1}\right)^{1/2} = \left(\frac{a^2}{\infty}\right)^{1/2} = 0$$

2.11.)



$f_r = kmv^2$

$$F = F_{gravity} - F_{friction}$$

$$ma = mg\sin\theta - kmv^2$$

$$v' = g\sin\theta - kv^2$$

$$\frac{dv}{dt} = k\left(\frac{g\sin\theta}{k} - v^2\right)$$

$$\frac{dv}{\left(\frac{g\sin\theta}{k} - v^2\right)} = k\,dt$$

$v_0 = 0$

$$\frac{1}{2\sqrt{\frac{g\sin\theta}{k}}} \ln\left|\frac{v + \sqrt{\frac{\sin\theta\, g}{k}}}{v - \sqrt{\frac{\sin\theta\, g}{k}}}\right| + \overset{0}{c} = kt$$

let $u = \sqrt{\frac{g\sin\theta}{k}}$

(here $t = f(v)$ want to find $v = f(t)$:)

$$\ln\left|\frac{v+u}{v-u}\right| = 2ukt$$

$$\left|\frac{v+u}{v-u}\right| = e^{2ukt}$$

Since $v_0 = 0$ and v is always positive by convention and since $|v-u|$ is in denominator and can't be zero we can assume $|v-u| = u - v$

$$v + u = (u - v)e^{2ukt}$$

$$v(1 + e^{2ukt}) = u(-1 + e^{2ukt})$$

$$v = u\frac{(-1 + e^{2ukt})}{(1 + e^{2ukt})}$$

now $v = f(t)$
rearrange for easy integration

$$v = \frac{u(-1 + e^{2ukt})(e^{-ukt})}{(1 + e^{2ukt})(e^{-ukt})}$$

$$v = \frac{u(e^{ukt} - e^{-ukt})}{(e^{ukt} + e^{-ukt})}$$

$$v = u \tanh(ukt)$$

$$\int dx = \int u \tanh(ukt)\, dt$$

$$x = \frac{u}{uk} \ln|\cosh(ukt)| + \cancel{C}^{\,0}$$

at $t=0$
$\ln|\cosh(0)| = \ln 1 = 0$
$\therefore\ C = x(0) = 0$

$$e^{kx} = \cosh(ukt)$$

$$ukt = \cosh^{-1}(e^{kx})$$

$$t = \frac{\cosh^{-1}(e^{kx})}{uk}$$

Substitute $u = \sqrt{\frac{g \sin\theta}{k}}$

$$t = \frac{\cosh^{-1}(e^{kx})}{k\sqrt{\frac{g\sin\theta}{k}}}$$

$$\boxed{t = \frac{\cosh^{-1}(e^{kx})}{\sqrt{kg\sin\theta}}}$$

2.17.) $\vec{F} = e\vec{E} + \frac{e}{c}(\vec{v} \times \vec{B})$

a.) given $\vec{E} = 0 \quad \vec{v} \perp \vec{B}$

$\therefore \quad \vec{F} = \frac{e}{c} v B \cancel{\sin 90^\circ}^{1}$

$m\dot{v} = \frac{e}{c} v B$ F is a vector!

$\dot{v} = \frac{eB}{mc} v$

$\int \frac{dv}{v} = \int \frac{eB}{mc} dt$

$\ln|v| - \ln|c| = e^{\frac{eB}{mc}t}$

$v = v_0 e^{\frac{eB}{mc}t}$ $v = \frac{dr}{dt}$

$\int dx = \int v_0 e^{\frac{eB}{mc}t} dt$

$r = \left(\frac{mc}{eB}\right) v_0 e^{\frac{eB}{mc}t}$

$r = \left(\frac{mc}{eB}\right) v$ let $\omega_c \equiv \frac{eB}{mc}$ the cyclotron frequency

$r = \frac{v}{\omega_c}$

b.) let $\vec{B} = B\hat{e}_z \qquad \vec{E} = E_y\hat{e}_y + E_z\hat{e}_z$

$$\vec{F} = e\left(E_y\hat{e}_y + E_z\hat{e}_z\right) + \frac{e}{c}\left(\vec{v}\times B\hat{e}_z\right)$$

$$\hat{e}_z\cdot\vec{F} = \hat{e}_z\cdot\left(eE_y\hat{e}_y + eE_z\hat{e}_z\right) + e_z\cdot\left(\frac{e}{c}\vec{v}\times B\hat{e}_z\right)$$

$$m\dot{v}_z = eE_z + \frac{e}{c}\vec{v}\cdot\left(\overset{0}{\cancel{B\hat{e}_z\times\hat{e}_z}}\right)$$

$$m\dot{v}_z = eE_z$$

$$\int dv_z = \int \frac{eE_z}{m}\,dt$$

$$v_z = \left(\frac{eE_z}{m}\right)t + C \qquad C = v_{0z} = \dot{z}_0 = \dot{z}(0)$$

$$\int dz = \int\left[\left(\frac{eE_z}{m}\right)t + v_{0z}\cancel{t}\right]dt$$

$$z = \left(\frac{eE_z}{2m}\right)t^2 + v_{0z}t + C' \qquad C' = z_0 = z(0)$$

$$z = \left(\frac{eE_z}{2m}\right)t^2 + \dot{z}t + z_0$$

(c.) $\vec{F} = e(E_y\hat{e}_y + E_z\hat{e}_z) + \frac{e}{c}\vec{v} \times B\hat{e}_z$

$$\hat{e}_y \cdot \vec{F} = e_y \cdot (eE_y\hat{e}_y + eE_z\hat{e}_z) + \frac{Be}{c}(e_y \cdot \vec{v} \times \hat{e}_z)$$

$$m\dot{v}_y = eE_y + \frac{Be}{c}\vec{v} \cdot \hat{e}_y \times \hat{e}_z$$

$$m\dot{v}_y = eE_y + \frac{Be}{c}\vec{v} \cdot (+\hat{e}_x)$$

$$m\dot{v}_y = eE_y + \frac{Be}{c}(+v_x)$$

$$\boxed{\dot{v}_y = \frac{eE_y}{m} + \frac{Be}{mc}(+v_x)} \quad ①$$

$$\vec{F} = e(E_y\hat{e}_y + E_z\hat{e}_z) + \frac{e}{c}\vec{v} \times B\hat{e}_z$$

$$\hat{e}_x \cdot \vec{F} = \hat{e}_x \cdot \cancel{(eE_y\hat{e}_y + eE_z\hat{e}_z)}^{\,0} + \frac{Be}{c}\hat{e}_x \cdot (\vec{v} \times \hat{e}_z)$$

$$m\dot{v}_x = \frac{Be}{c}\vec{v} \cdot (\hat{e}_x \times \hat{e}_z)$$

$$\boxed{\dot{v}_x = \frac{Be}{mc}(-v_y)} \quad ②$$

now use these two equations to find v_y v_x

$$-v_y\frac{Be}{mc} = \dot{v}_x \quad ②$$

$$\dot{v}_y = -\frac{mc}{Be}\ddot{v}_x$$ substitute $\dot{v}_y$ into ① to find v_x

$$-\frac{mc}{Be}\dddot{v}_x = \frac{eE_y}{m} + \frac{Be}{mc}v_x$$

$$\ddot{v}_x + \left(\frac{Be}{mc}\right)\left(\frac{Be}{mc}\right)v_x = -\left(\frac{eE_y}{m}\right)\left(\frac{Be}{mc}\right)$$

$$\left[D^2 + \left(\frac{Be}{mc}\right)^2\right]v_x = -\left(\frac{eE_y}{m}\right)\left(\frac{Be}{mc}\right)$$

$$\lambda^2 + \left(\frac{Be}{mc}\right)^2 = 0 \Rightarrow \lambda = \pm i\left(\frac{Be}{mc}\right) \qquad \text{let } \omega_c = \frac{Be}{mc}$$

$$v_{xH} = A\sin\omega_c t + B\cos\omega_c t$$

find particular solution: $r(t) = \left(-\frac{eE_y}{m}\right)\omega_c$

Wronskian $= W = -y_1 y_2' - y_1' y_2$

$$W = -\omega_c \sin^2\omega_c t - \omega_c\cos^2\omega_c t = -\omega_c$$

$$y_p = -y_1\int \frac{y_2 r\,dt}{W} + y_2\int\frac{y_1 r}{W}dt$$

$$y_p = +\sin\omega_c t\int\frac{(\cos\omega_c t)}{-\omega_c}\left(-\frac{eE_y}{m}\right)\omega_c\,dt - \cos\omega_c t\int\frac{\sin(\omega_c t)}{-\omega_c}\left[\frac{eE_y}{m}\right]\omega_c dt$$

$$y_p = +\sin\omega_c t\left(\frac{eE_y}{m}\right)\frac{\sin\omega_c t}{\omega_c} + \left(\frac{eE_y}{m}\right)(\cos\omega_c t)\frac{\cos\omega_c t}{\omega_c}$$

$$y_p = +\left(\frac{eE_y}{m}\right)\frac{1}{\omega_c}$$

$$v_x = +\frac{eE_y}{\omega_c m} + A\sin\omega_c t + B\cos\omega_c t$$

$$\langle v_x\rangle = \frac{\int_0^{\frac{2\pi}{\omega_c}}\left(\frac{eE_y}{\omega_c m} + A\sin\omega_c t + B\cos\omega_c t\right)dt}{\int_0^{\frac{2\pi}{\omega_c}}dt} = \frac{\frac{eE_y}{\omega_c m}\left(\frac{2\pi}{\omega_c}\right) + 0 + 0}{\frac{2\pi}{\omega_c}}$$

$$\langle \dot{v}_x \rangle = \frac{eE_y}{\omega_c m} = \frac{eE_y}{m}\left(\frac{mc}{Be}\right) = \frac{cE_y}{B}$$

$$\boxed{\therefore \langle \dot{x} \rangle = \frac{cE_y}{B}}$$

now find $\langle \dot{y} \rangle$ go back to equation ①

$$\dot{v}_y = \frac{eE_y}{m} + \frac{Be}{mc} v_x$$

$$v_x = \frac{\dot{v}_y}{\omega_c} + \frac{eE_y}{\omega_c m}$$

$$\dot{v}_x = \frac{\ddot{v}_y}{\omega_c}$$ substitute into equation ②

$$\frac{\ddot{v}_y}{\omega_c} = -\omega_c v_y$$

$$\ddot{v}_y = -\omega_c^2 v_y$$

$$(D^2 + \omega_c^2) v_y = 0$$

$$\lambda^2 + \omega_c^2 = 0 \Rightarrow \lambda = \pm i\omega_c$$

$$v_y = c_1 e^{i\omega_c t} + c_2 e^{-i\omega_c t} = c_2' \sin \omega_c t + c_1' \cos \omega_c t$$

$$\langle v_y \rangle = \frac{\int_0^{2\pi/\omega_c} c_2 \sin \omega_c t + c_1 \cos \omega_c t \, dt}{\int_0^{2\pi/\omega_c} dt}$$

$$\langle v_y \rangle = \frac{\omega_c}{2\pi}\left[-\frac{c_2}{\omega_c} \cos \omega_c t \Big|_0^{2\pi/\omega_c} + \frac{c_1}{\omega_c} \cos \omega_c t \Big|_0^{2\pi} \right] = 0$$

$$\boxed{\therefore \langle \dot{y} \rangle = 0}$$

d.) I think there's a misprint in book $\omega_L = \omega_c$?

$$\frac{dy}{dt} = C_1 \sin\omega_c t + C_2 \cos\omega_c t$$

$$v_y(0) = C_2$$

$$y = \int C_1 \sin\omega_c t + v_{oy} \cos\omega_c t \, dt$$

$$y = -\frac{C_1}{\omega_c}\cos\omega_c t + \frac{v_{oy}}{\omega_c}\sin\omega_c t + C$$

$$y(0) = -\frac{C_1}{\omega_c} + C = 0 \qquad C = \frac{C_1}{\omega_c}$$

$$y(t) = -\frac{C_1}{\omega_c}\left(\cos\omega_c - 1\right) + \frac{v_{oy}}{\omega_c}\sin\omega_c t$$

let $v_{oy} = 0$ and $A = -C_1 = \text{const.}$

$$\boxed{y(t) = \frac{A}{\omega_c}\left(\cos\omega_c - 1\right)}$$

$$\frac{dx}{dt} = C_1 \sin\omega_c t + C_2 \cos\omega_c t + \frac{cE_y}{B}$$

$$v_x(0) = C_2 + \frac{cE_y}{B}$$

$$x = -\frac{C_1}{\omega_c}\cos\omega_c t + \frac{C_2}{\omega_c}\sin\omega_c t + \frac{cE_y}{B}t$$

$$x(0) = -\frac{C_1}{\omega_c} = 0 \quad \therefore \quad C_1 = 0$$

$$\boxed{x(t) = \frac{C_2}{\omega_c}\sin\omega_c t + \frac{cE_y}{B}t}$$

where $v(0) = C_2 + \frac{cE_y}{B}$

2.22) a particle falls to Earth from rest at a great height. neglect air resistance – show that it takes approx: 9/11 total time to fall the first ½ distance.

$h = \frac{gt^2}{2}$ $\quad h \downarrow \quad \downarrow h/2 \quad$ in constant g field

$t = \sqrt{\frac{2h}{g}}$

$$T_{ratio} = \frac{t(h/2)}{t(h)} = \frac{\sqrt{\frac{2h}{2g}}}{\sqrt{\frac{2h}{g}}} = \frac{1}{\sqrt{2}} \sim 0.707$$

since $\frac{9}{11} \approx 0.8182 \not\approx 0.707$ ∴ we can't assume $g = $ const.

$$\vec{F} = \vec{F}$$

$$m\ddot{r} = \frac{\gamma m M}{r^2} \qquad \frac{dv}{dt} = \frac{dv}{dr}\frac{dr}{dt} = \frac{v\,dv}{dr}$$

$$\frac{v\,dv}{dr} = \frac{\gamma M}{r^2}$$

$$\int v\,dv = \int \frac{\gamma M\,dr}{r^2}$$

$$\frac{v^2}{2} = \frac{\gamma M}{r^2} + C$$

$$v = \left(\frac{2\gamma M}{r} + C'\right)^{1/2}$$ starts at rest $v = 0$ at $h = r$ ← a great height

$$0 = \left(\frac{2\gamma M}{r} + C'\right)^{1/2}$$

$$C' = -\frac{2\gamma M}{h}$$

$$v = \left(\frac{2\gamma M}{r} - \frac{2\gamma M}{h}\right)^{1/2}$$

$$v = (2\gamma M)^{1/2}\left(\frac{1}{r} - \frac{1}{h}\right)^{1/2} = (2\gamma M)^{1/2}\left(\frac{h-r}{rh}\right)^{1/2}$$

$$\left(\frac{rh}{h-r}\right)^{1/2} dr = (2\gamma M)^{1/2} t$$

$$\int_{r_i}^{r_f}\left(\frac{rh}{h-r}\right)^{1/2} dr = \int_0^t (2\gamma M)^{1/2} t$$

let $r = h\cos^2\theta$
$dr = -2h\cos\theta\sin\theta\, d\theta$
$\theta = \cos^{-1}\sqrt{\frac{r}{h}}$

$$\int_{\cos^{-1}\sqrt{\frac{r_i}{h}}}^{\cos^{-1}\sqrt{\frac{r_f}{h}}} \frac{(h^2\cos^2\theta)^{1/2}(-2h\cos\theta\sin\theta\, d\theta)}{(h - h\cos^2\theta)^{1/2}} = \int_0^t (2\gamma M)^{1/2} t$$

$$\int_{\cos^{-1}\sqrt{r_i/h}}^{\cos^{-1}\sqrt{r_f/h}} \frac{-2h^2\cos^2\theta\, \cancel{\sin\theta}\, d\theta}{h^{1/2}(\cancel{1-\cos^2\theta})^{1/2}} = (2\gamma M)^{1/2} t$$

$$\int_{\cos^{-1}\sqrt{r_i/h}}^{\cos^{-1}\sqrt{r_f/h}} -2h^{3/2}\cos^2\theta\, d\theta = (2\gamma M)^{1/2} t$$

$$-2h^{3/2}\left[\frac{1}{2}\sin\theta\cos\theta + \frac{1}{2}\theta\right]_{\cos^{-1}\sqrt{r_i/h}}^{\cos^{-1}\sqrt{r_f/h}} = (2\gamma M)^{1/2} t$$

$$T_{ratio} = \frac{t(h/2)}{t(h)} = \frac{-\left[\frac{1}{2}\sin\theta\cos\theta + \frac{1}{2}\theta\right]\Big|_{\cos^{-1}\sqrt{1} = 0}^{\cos^{-1}\sqrt{\frac{1}{2}} = \pi/4}}{-\left[\frac{1}{2}\sin\theta\cos\theta + \frac{1}{2}\theta\right]\Big|_{\cos^{-1}\sqrt{1} = 0}^{\cos^{-1}\sqrt{r_e/h} \sim \cos^{-1}\sqrt{0}\ \pi/2}} \qquad \frac{r_e}{h} \sim 0$$

$$\cancel{T = \frac{-\left[\frac{1}{4} + \frac{\pi}{8} - 0 - \frac{\pi}{4}\right]}{-\left[0 + 0 - 0 - \pi/4\right]} = \frac{1}{2} + \frac{1}{\pi} = 0.8183 \sim \frac{9}{11} = .81\overline{81}}$$

$$T_{ratio} = \frac{\left[\frac{1}{2}\sin\theta\cos\theta + \frac{1}{2}\theta\right]\Big|_{\pi/4}^{0}}{\left[\frac{1}{2}\sin\theta\cos\theta + \frac{1}{2}\theta\right]\Big|_{\pi/2}^{0}}$$

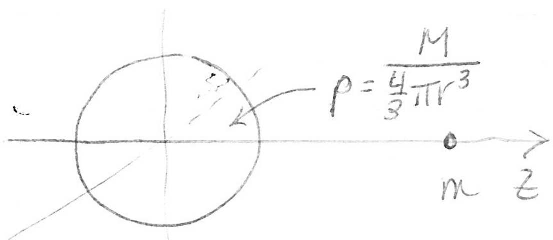
$$T = \frac{\left[0 + 0 - \left(\frac{1}{4} + \frac{\pi}{8}\right)\right]}{\left[0 + 0 - \left(0 + \frac{\pi}{4}\right)\right]}$$

$$T = \frac{1}{2} + \frac{1}{\pi} = 0.8183$$

$$\frac{9}{11} \sim 0.8181\overline{81}$$

$$\therefore T \sim \frac{9}{11}$$

2.23.) Gravitational force on a unit mass at a point <u>exterior</u> to homogeneous sphere of matter.



$$R^2 = z^2 + r'^2 - 2zr'\cos\theta'$$

$$\vec{F} = -\gamma\rho m \int_{\text{sphere}} \frac{(z\hat{z} - r'\hat{r}')\,d\tau}{(z^2 + r'^2 - 2zr'\cos\theta')^{3/2}}$$

$$\vec{F} = -\gamma\rho m \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \frac{(z - r'\cos\theta')r'^2\sin\theta'\,dr'\,d\theta'\,d\phi'}{(z^2 + r'^2 - 2zr'\cos\theta')^{3/2}}$$

$$\vec{F} = -\gamma\rho m\, 2\pi \int_0^a r'^2\,dr' \int_{-1}^{+1} \frac{z - r'\mu\,d\mu}{(z^2 + r'^2 - 2zr'\mu)^{3/2}}$$

$$\vec{F} = -\gamma\rho m\, 2\pi \int_0^a r'^2 dr' \left[\frac{1}{z^2}\right]\left[\frac{z-r'}{|z-r'|} + \frac{z+r'}{|z+r'|}\right] \quad \text{for } z > a$$

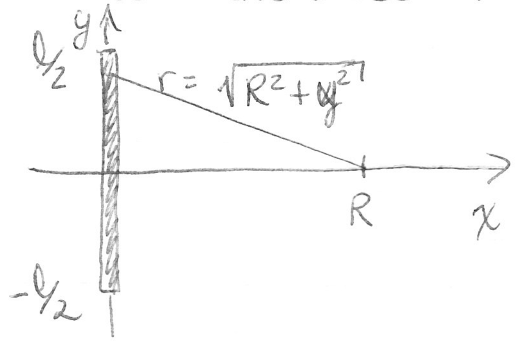
$$\vec{F} = -\gamma\rho m\, 4\pi \int_0^a r'^2 dr' \left(\frac{2}{z^2}\right)$$

$$\vec{F} = -\frac{\gamma\rho m 4\pi 2}{z^2}\left(\frac{r^3}{3}\right)\Big|_0^a = -\frac{\gamma\rho m}{z^2}2\left(\frac{4\pi a^3}{3}\right)$$

Substitute $M = \rho\left(\frac{4\pi a^3}{3}\right)$

$$\boxed{\vec{F} = -\frac{\gamma m M}{z^2}}$$

2.25.) calculate gravitational potential due to thin rod of length l mass R at distance R $\perp$ to center.



$M = \rho l$

$$\phi = -\gamma \int_L \frac{\rho}{r} ds' \qquad ds' = dy$$

$$\phi = -\gamma \int_{-l/2}^{l/2} \frac{\rho \, dy}{\sqrt{R^2+y^2}} \qquad \rho = \text{cnst} = \frac{M}{l}$$

$$\phi = -\frac{\gamma M}{l} 2 \int_0^{l/2} \frac{dy}{\sqrt{R^2+y^2}}$$

$$\phi = -\frac{2\gamma M}{l} \left[\ln\left(y + \sqrt{y^2+R^2}\right) \right] \Big|_0^{l/2}$$

$$\boxed{\phi = -\frac{\gamma 2M}{l} \ln \left[\frac{l/2 + \sqrt{(l/2)^2 + R^2}}{R} \right]}$$