Quant Mech Meg Noah

$9\frac{1}{2}/10$

18.2.1 $H'(t) = \dfrac{-e\mathcal{E}X}{[1+(t/\tau)^2]}$

$$d_n(\infty) = \frac{-i}{\hbar}\int_{-\infty}^{+\infty} (-e\mathcal{E})\langle n|X|0\rangle \frac{1}{[1+(t/\tau)^2]} e^{in\omega t}\,dt$$

$$X = \left(\frac{\hbar}{2m\omega}\right)^{1/2}(a+a^\dagger) \qquad a^\dagger|0\rangle = |1\rangle$$

$$d_1(\infty) = \frac{ie\mathcal{E}}{\hbar}\left(\frac{\hbar}{2m\omega}\right)^{1/2}\int_{-\infty}^{+\infty}\frac{e^{i\omega t}}{(1+(t/\tau)^2)}dt = \frac{ie\mathcal{E}}{\hbar}\left(\frac{\hbar}{2m\omega}\right)^{1/2}\tau^2\int_{-\infty}^{+\infty}\frac{e^{i\omega t}dt}{(\tau^2+t^2)}$$

integrate by parts $\int u\,dv = uv - \int v\,du$

$dv = e^{i\omega t}dt \qquad V = \dfrac{e^{i\omega t}}{i\omega}$

$u = (1+(t/\tau)^2)^{-1} \qquad du = \dfrac{-2(t/\tau^2)dt}{(1+(t/\tau)^2)^2}$

$$\int_{-\infty}^{+\infty}\frac{e^{i\omega t}}{(1+(t/\tau)^2)}dt = \left.\frac{e^{i\omega t}}{i\omega(1+(t/\tau)^2)}\right|_{-\infty}^{+\infty} - \int\frac{e^{i\omega t}}{i\omega}$$

Use Cauchy Residue Theorem.

Wolfram Integrator ~~$a=i\omega$ $b=\tau$~~ $a=\omega$ $b=\tau$

~~$\int [e^{ax}/((\frac{x}{b})^2+1)]dx = -\frac{1}{2}ibe^{-ibx}(e^{2iab}Ei(a(x-ib)) - Ei(a(ib+x)))$~~

$$\int_{-\infty}^{+\infty}\left(\frac{e^{iax}}{b^2+x^2}\right)dx = \left.\frac{ie^{-ab}(e^{2ab}Ei(iax-ab) - Ei(a(b+ix)))}{2b}\right|_{-\infty}^{+\infty} = \frac{i}{b}e^{-ab}\pi$$

$$d_1(\infty) = \frac{ie\mathcal{E}}{\hbar}\left(\frac{\hbar}{2m\omega}\right)^{1/2}\tau^2\left[\frac{ie^{-\omega\tau}\pi}{\tau}\right]$$

$$P_{0\to1} = |d_1(\infty)|^2 = \frac{e^2\mathcal{E}^2}{\hbar^2}\frac{\hbar}{2m\omega}\frac{\tau^4}{\tau^2}e^{-2\omega\tau}\pi^2 = \frac{\pi^2e^2\mathcal{E}^2\tau^2e^{-2\omega\tau}}{2m\omega\hbar}$$

18.2.2 $\vec{E}(t) = \mathcal{E}e^{-t^2/\tau^2}\hat{k} \Rightarrow V = -e\mathcal{E}r\cos\theta e^{-t^2/\tau^2}$

$\hat{k} = z\hat{z}$

$$d = -\frac{i}{\hbar}\int_{-\infty}^{+\infty}dt\,e^{i\omega t}e^{-t^2/\tau}(-e\mathcal{E})\langle 2lm|r\cos\theta|100\rangle \qquad r\cos\theta = \sqrt{\frac{3}{4\pi}}Y_{10}(\theta,\phi)r$$

$\therefore$ only $l=1$ $m=0$ is non orthogonal to $z|100\rangle$

$\langle 210|r\cos\theta|100\rangle =$

$$= \int dr\,r^2\int_{-1}^{+1}d\cos\theta\int_0^{2\pi}d\phi\,R_{21}(r)Y_{10}^*(\theta)r\cos\theta R_{10}(r)Y_{00}$$

$$= \frac{\sqrt{3}}{4\pi}2\pi\int_{-1}^{1}dx\,x^2\int_0^\infty dr\,r^3R_{21}(r)R_{10}(r) \quad \text{where } x\leftrightarrow\cos\theta$$

$$= \frac{1}{\sqrt{3}}\int_0^\infty dr\,r^3R_{21}(r)R_{10}(r) = \frac{1}{\sqrt{3}}\int_0^\infty dr\,r^2\frac{1}{\sqrt{24}}\frac{1}{a_0^{5/2}}r\,e^{-r/2a_0}\,r\frac{2}{a_0^{3/2}}e^{-r/a_0} \rightarrow$$

$$= \frac{1}{\sqrt{2}} \int_0^\infty dr\, r^4 e^{-3r/2a_0} \frac{1}{a_0^4} = \frac{1}{\sqrt{2}} \frac{4!}{a_0^4} \left(\frac{2a_0}{3}\right)^5 = \sqrt{2} \frac{2^7}{3^5} a_0$$

$$\therefore\ d = \frac{-i}{\hbar} \sqrt{2}\, \cancel{} \frac{2^7}{3^5} a_0 (-e\mathcal{E}) \underbrace{\int_{-\infty}^{+\infty} dt\, e^{i\omega t} e^{-t^2/\tau^2}}_{\sqrt{\pi}\,\tau\, e^{(\omega\tau/2)^2}} = \frac{-i}{\hbar} \frac{2^{5/2}}{3^5} a_0 (-e\mathcal{E}) \sqrt{\pi}\,\tau\, e^{(\omega\tau/2)^2}$$

$$P(n=1 \to n=2) = |d|^2 = \frac{e^2 \mathcal{E}^2 a_0^2}{\hbar^2} \frac{2^{15}}{3^{10}} \pi \tau^2 e^{-\omega^2\tau^2/2}$$

Spin matters in magnetic field is applied.

18.2.3 Cargue

$\frac{p}{m} = V \qquad VT \propto L \qquad p = \sqrt{2mE} \qquad E_n = \hbar^2\pi^2 n^2/2mL^2$

$p = \sqrt{2m\hbar^2\pi^2 n^2/2mL^2} = \hbar\pi n/L$

$\Rightarrow T_n \sim 2L^2 m / \hbar\pi n$

$$\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \qquad E_n = \frac{\hbar^2\pi^2}{2mL^2} n^2 = \frac{\hbar^2\pi^2}{2mL^2}$$

$H_t = \frac{p^2}{2m}$ ~~Enew~~ the box $L \to 2L$ ~~E~~

~~state is < $\hbar^2\pi^2$~~

$$E_{n\,new} = \frac{\hbar^2\pi^2}{2m(4L^2)} n^2 = \frac{\hbar^2\pi^2}{8mL^2} n^2 = \frac{\hbar^2\pi^2}{2mL^2} \Rightarrow n^2 = 4 \therefore n = 2$$

on energy arguments
the particle is now in the $n=2$ state

$P_{2\to1} = ?$ $\qquad H' = H^0$

$$d_f(t) = \delta_{fi} - \frac{i}{\hbar} \int_0^t \langle f^0 | H' | i^0 \rangle e^{i\omega_{fi} t} dt \Rightarrow \cancel{\delta_{21}}^{0} - \frac{i}{\hbar} \int_0^t \langle 1 | H | 2 \rangle e^{i\omega_{21} t} dt = 0$$

the probability it is in ground state is zero.

1/2

18.2.3 $E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2} \Rightarrow P_n = \sqrt{2mE_n} \Rightarrow p = \frac{\hbar \pi n}{L}$

"argue" $T_n \sim \frac{2Lm}{p_n} = \frac{2L^2 m}{\hbar \pi n}$ $\quad T_1 = \frac{2L^2 m}{\hbar\pi} \sim \frac{L^2 m}{\hbar \pi}$

$\tau << T \therefore$ "sudden"

$|\psi_{0\,before}\rangle = \sqrt{\frac{2}{L}} \cos\frac{\pi x}{L}$ $\quad |\psi_{0\,after}\rangle = \sqrt{\frac{1}{L}} \cos\frac{\pi x}{2L}$

$$P = |\langle \psi_{0\,before} | \psi_{0\,after}\rangle|^2 = \left| \frac{\sqrt{2}}{L} \int_{-L/2}^{L/2} dx \cos\frac{\pi x}{2L} \cos\frac{\pi x}{L} \right|^2$$

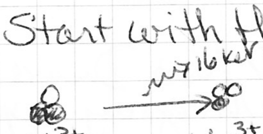
$$= \frac{2}{L^2} \left| \int_{-L/2}^{+L/2} dx \, \tfrac{1}{2}\left(\cos\frac{\pi x}{2L} + \cos\frac{3\pi x}{2L}\right)\right|^2 = \frac{2}{L}\frac{1}{4} \left| 2 \int_0^{L/2} dx \left(\cos\frac{\pi x}{2L} + \cos\frac{3\pi x}{2L}\right)\right|^2$$

$$= \frac{2}{L^2} \left| \frac{2L}{\pi} \sin\frac{\pi}{2L}\frac{L}{2} + \frac{2L}{3\pi} \sin\frac{3\pi}{2L}\frac{L}{2} \right|^2 = \frac{2}{L^2}\left| \frac{2L}{\pi} \sin\frac{\pi}{4} + \frac{2L}{3\pi} \sin\frac{3\pi}{4}\right|^2$$

$\frac{1}{2}$

$$= \frac{2}{L^2} \left| \frac{2L}{3\pi} \left(\frac{3}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right|^2 = \frac{2}{L^2} \frac{4L^2}{9\pi^2} (2\sqrt{2})^2 = \frac{64}{9\pi^2} \approx .72$$

18.2.4 Start with Hydrogen



$n=1 \; l=0 \; m=0$

$|100\rangle = |\psi_{before}\rangle = \left(\frac{1}{\pi a_0^3}\right)^{1/2} e^{-r/a_0}$ $\quad E_1 = -13.6\,eV$

for ground state He $a_0 \rightarrow \frac{\hbar^2}{m(2e)^2} = \frac{\hbar^2}{4me^2} = \frac{a_0}{4}$

$\psi_{after} = \left(\frac{64}{\pi a_0^3}\right)^{1/2} e^{-4r/a_0}$

$$1 = \int \psi_{after}^* \psi_{after} \, r^2 d\Omega = A^2 4\pi \int_0^{4a_0} r^2 e^{-8r/a_0} dr = A^2 4\pi \frac{e^{-8r/a_0}}{(-8/a_0)}\left(r^2 - \frac{2r}{(-8/a_0)} + \frac{2}{(-8/a_0)^2}\right)\Big|_0^{4a_0}$$

$$1 = A^2 4\pi \left[e^{-2}(\ldots) + \frac{a_0}{8}\left(a_0^2 + \frac{16^2 a_0^2}{8} - \frac{2a_0^2}{64}\right) e^{-8} \right] = a_0^3 \pi \left(\frac{27}{16\sqrt{2}}\right)^2$$

$$\Rightarrow A = \frac{27}{16\sqrt{2}} \left(\frac{1}{\pi a_0^3}\right)^{1/2}$$

$\psi_{after} = |16, 3, 0\rangle =$

$P = |\langle \psi_{before} | \psi_{after}\rangle|^2 = |\langle 1,0,0 | 16,3,0\rangle|^2 = 0 \quad \Delta l \neq 0, \pm 1$

$\cancel{\frac{1}{2(2\pi)}\int A_1^* A_2}$

18.2.5 $\quad H = H^0 + H' \qquad H' = -fx$

at $t=0$ H' is turned off

before we found the eigenftn's are the same for Harmonic oscillator with + without $-fx$ applied

$$|\psi_{before}\rangle = |n\rangle = |\psi_{after}\rangle$$

$$|n\rangle = |n_0\rangle + \sum_m |m^0\rangle\langle m^0| \frac{-qf(\hbar/2m\omega)^{1/2}(a+a^\dagger)|n^0\rangle}{E_n^0 - E_m^0}$$

$$= |n^0\rangle + qf\left(\frac{1}{2\hbar m\omega^3}\right)^{1/2}\left[(n+1)^{1/2}|(n+1)^0\rangle - n^{1/2}|(n-1)^0\rangle\right]$$

~~after $|n\rangle =$~~

$$H_{after} = H^0$$

found in a paper PRA Vol 47 No3 Driven Harmonic Oscillators in the Adiabatic Magnus Approx

Klarfeld & ~~Oteo~~ $\quad \cancel{V=-f}\quad V_1(t) = f(t)x = -fx$

$$U^I(t,-\infty) = \exp[\Omega_1(t) + \Omega_2(t)] \qquad \sigma_0^2 = \cancel{\quad}\langle x^2\rangle = \langle 0|x^2|0\rangle = \left(\frac{\hbar}{2m\omega_0}\right)$$

where $\Omega_1(t) = -\frac{i}{\hbar}\int_{-\infty}^{t} dt_1 H_1^I = -i\xi(t)a^\dagger - i\xi^*(t)a$

$$\Omega_2 = -\frac{1}{2\hbar^2}\int_{-\infty}^{t} dt_2 \int_{-\infty}^{t_2} dt_1 [H_2^I, H_1^I] = -i\eta(t)$$

$$\xi = \frac{\sigma_0}{\hbar}\int_{-\infty}^{t} dt_1 f(t_1) e^{i\omega_0 t} \qquad \eta = \frac{\sigma_0^2}{2\hbar^2}\int_{-\infty}^{t} dt_2 f(t_2)\int_{-\infty}^{t_2} dt_1 f(t_1)\sin\omega_0(t_1-t_2)$$

$$P_{0\to n}(t) = |\langle n|U^I(t,-\infty)|0\rangle|^2 = |\xi|^{2n} e^{-|\xi|^2}/n! \Rightarrow \frac{\lambda^n e^{-\lambda}}{n!} \qquad \lambda = \frac{f^2}{2m\omega^3\hbar}$$

$$\xi = \frac{1}{\hbar}\left(\frac{\hbar}{2m\omega_0}\right)^{1/2}\int_{-\infty}^{t_1} dt_1(-f)e^{i\omega_0 t} = \frac{f}{\cancel{2m\omega_0}(2m\omega^2\hbar)^{1/2}} \qquad \text{via } \xi = \frac{f}{(2m\omega^2\hbar)^{1/2}}$$

$f(t_1) = -f$

18.2.6 $H'(t) = H'\delta(t)$

$t = 0^- \quad |i^0\rangle \quad , \quad |f^0\rangle$ at $t = 0^+$

$$d_f = \delta_{fi} - \frac{i}{\hbar}\int_0^{i} \langle f^0|H'(t')|i^0\rangle e^{i\omega_{fi}t'}dt'$$

$$= \delta_{fi} - \frac{i}{\hbar}\int_0^{i} \langle f^0|H'\delta(t)|i^0\rangle e^{i\omega_{fi}t}dt$$

$f \neq i$

$$= -\frac{i}{\hbar}\int_0^{i} \delta(t)\langle f^0|H'|i^0\rangle e^{i\omega_{fi}t}dt$$

$$= -\frac{i}{\hbar}\langle f^0|H'|i^0\rangle \cancel{e^{0}}^{1} = \frac{-i}{\hbar}\langle f^0|H'|i^0\rangle$$

Couldn't finish — had flu!