10 m Dipole Antenna Radiation Resistance

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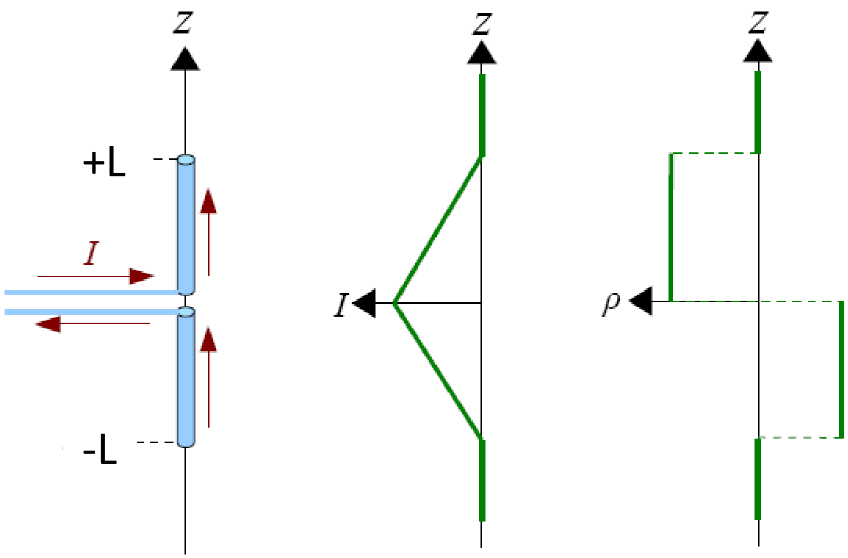
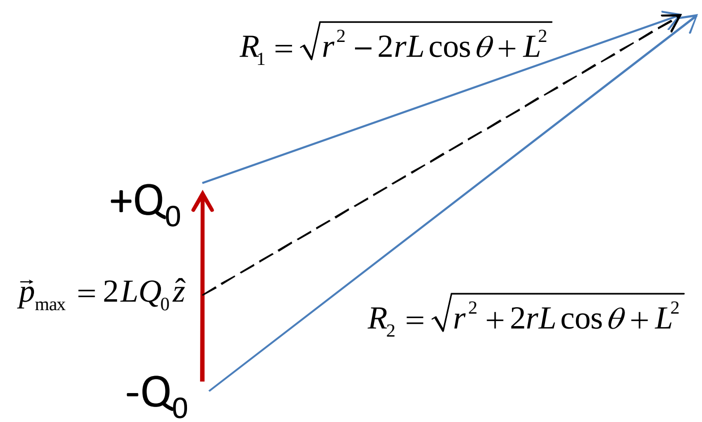
Antenna Operating at ω

- Source current – in these derivations it is equivalent to an oscillating point dipole



$$I(z,t) = I_0\left(1 - \frac{2|z|}{(2L)}\right)e^{-i\omega t}$$

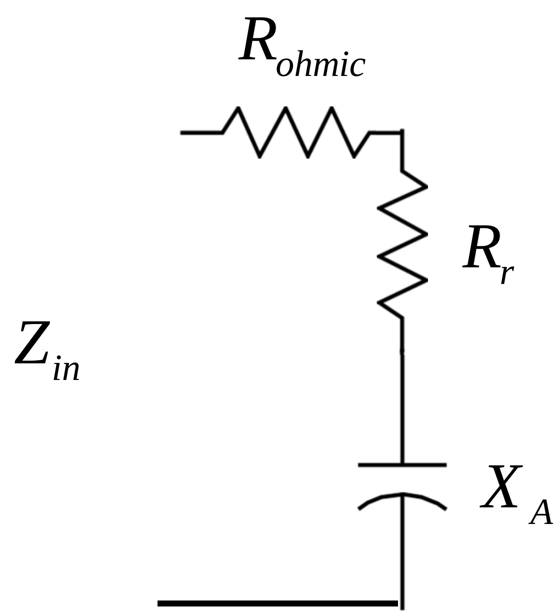


$$Q(t) = Q_0\cos(\omega t)$$

Input Impedance

- Want to know how much power is radiated to a field point for a given current.

$$Z_{in} = R_{ohmic} + R_r - jX_A$$



Scalar Potential

$$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{\rho_C(\vec{r}\,')\,d\tau'}{R} + \frac{1}{4\pi\varepsilon_0}\int_{A'}\frac{\sigma_C(\vec{r}\,')\,da'}{R} + \frac{1}{4\pi\varepsilon_0}\int_{L'}\frac{\lambda_C(\vec{r}\,')\,ds'}{R} + \frac{1}{4\pi\varepsilon_0}\sum_N \frac{q_i}{R} + C$$

$$\vec{r} = \rho\hat{\rho} + z\hat{z} \qquad \vec{r}\,' = z'\hat{z} \quad (\text{-L}<z'<\text{L}) \qquad R = \vec{r} - \vec{r}\,' = \rho\hat{\rho} + (z - z')\hat{z}$$

$$\phi(\rho, z) = \frac{\lambda}{4\pi\varepsilon_0}\int_{-L}^{+L}\frac{dz'}{\sqrt{\rho^2 + (z-z')^2}} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left\{\frac{z + L + \sqrt{\rho^2 + (z+L)^2}}{z - L + \sqrt{\rho^2 + (z-L)^2}}\right\}$$

$$\phi(\rho, z) \xrightarrow[\substack{L>>\rho \\ L>>z}]{} \frac{\lambda}{4\pi\varepsilon_0}\ln\left\{\frac{2L}{(L-z)\left(-1+\sqrt{1+\frac{\rho^2}{(z-L)^2}}\right)}\right\}$$

$$\phi(\rho, z) = \frac{\lambda}{4\pi\varepsilon_0}\ln\left\{\frac{2L}{L\left(-1+\sqrt{1+\left(\frac{\rho}{L}\right)^2}\right)}\right\} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left\{\frac{2}{\left(-1+1+\frac{1}{2}\left(\frac{\rho}{L}\right)^2\right)}\right\} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left\{\left(\frac{2L}{\rho}\right)^2\right\}$$

$$\phi(\rho, z) = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{2L}{\rho}\right)$$

$2L =$ antenna length

$a =$ antenna radius

Capacitance

$$C = \frac{Q}{\phi}$$

$$\Delta\phi(a,z) = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{2L}{a}\right) = \frac{dQ/dz}{2\pi\varepsilon_0}\ln\left(\frac{2L}{a}\right)$$

$$\frac{dC}{dz} = \frac{dQ}{d\phi} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{2L}{a}\right)}dQ$$

$$C = \frac{4\pi\varepsilon_0 L}{\ln\left(\frac{2L}{a}\right)}$$

Capacitive Reactance

$$X = -\frac{1}{\omega C}$$

$$C = \frac{4\pi\varepsilon_0 L}{\ln\left(\frac{2L}{a}\right)}$$

$$X_A = -\frac{1}{\omega C} = -\frac{\ln\left(\frac{2L}{a}\right)}{4\pi\varepsilon_0 \omega L}$$

$$\boxed{X_A = -\frac{Z_0}{4\pi \frac{\omega}{c} L} \ln\left(\frac{2L}{a}\right)}$$

$2L =$ antenna length

$a =$ antenna radius

Retarded Potentials

- Light from a distant star is observed at time t, the light left the star at t_d.

$$\phi(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\rho_C(\vec{r}\,',t_r)\,d\tau'}{R}$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int_{V'} \frac{\vec{J}(\vec{r}\,',t_r)\,d\tau'}{R}$$

Retarded Scalar Potential

- Regard dipole antenna as oscillating point dipole. This is exact:

$$Q(t) = Q_0 \cos(\omega t), \quad \vec{p}(t) = 2LQ_0 \cos(\omega t)\hat{z} = p_0 \cos(\omega t)\hat{z}$$

$$\phi(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\left\{\frac{Q_0 \cos(\omega(t - R_1/c))}{R_1} - \frac{Q_0 \cos(\omega(t - R_2/c))}{R_2}\right\}$$

$$R_1 = \sqrt{r^2 - 2rL\cos\theta + L^2} \qquad R_2 = \sqrt{r^2 + 2rL\cos\theta + L^2}$$

- Approximate for small separation

$2L \ll r$

$$\frac{1}{R_1} \sim \frac{1}{r}\left(1 + \frac{L}{r}\cos\theta\right) \quad \frac{1}{R_2} \sim \frac{1}{r}\left(1 - \frac{L}{r}\cos\theta\right)$$

$$\cos(\omega(t - R_1/c)) \sim \cos\left(\omega(t - r/c) + \frac{\omega L}{c}\cos\theta\right) = \cos(\omega(t - r/c))\cos\left(\frac{\omega L}{c}\cos\theta\right) - \sin(\omega(t - r/c))\sin\left(\frac{\omega L}{c}\cos\theta\right)$$

$$\cos(\omega(t - R_2/c)) \sim \cos\left(\omega(t - r/c) + \frac{\omega L}{c}\cos\theta\right) = \cos(\omega(t - r/c))\cos\left(\frac{\omega L}{c}\cos\theta\right) - \sin(\omega(t - r/c))\sin\left(\frac{\omega L}{c}\cos\theta\right)$$

Retarded Scalar Potential

- Approximate as a perfect dipole:

$$2L \ll \frac{c}{\omega}$$

$$\cos\left(\omega\left(t - R_1 / c\right)\right) \sim \cos\left(\omega\left(t - r / c\right)\right) - \frac{\omega L}{c} \cos\theta \sin\left(\omega\left(t - r / c\right)\right)$$

$$\cos\left(\omega\left(t - R_2 / c\right)\right) \sim \cos\left(\omega\left(t - r / c\right)\right) + \frac{\omega L}{c} \cos\theta \sin\left(\omega\left(t - r / c\right)\right)$$

- Approximate for far field.

$$\frac{c}{\omega} \ll r$$

$$\boxed{V\left(r, \theta, t\right) = -\frac{p_0}{4\pi\varepsilon_0} \frac{\omega}{c} \frac{\cos\theta}{r} \sin\left(\omega\left(t - r / c\right)\right)}$$

Retarded Vector Potential

- Current in antenna taken as oscillating point charges:

$$I(t) = \frac{dQ}{dt}\hat{z} = -Q_0\omega\sin(\omega t)\hat{z}$$

$$\vec{A}(r,t) = \frac{\mu_0}{4\pi}\int_{-d/2}^{d/2}\frac{-Q_0\omega\sin(\omega(t-r/c))}{r}\hat{z}$$

$$\vec{A}(r,t) = -\frac{\mu_0 p_0}{4\pi}\frac{\omega}{r}\sin(\omega(t-r/c))\hat{z}$$

$$\boxed{\vec{A}(r,t) = -\frac{\mu_0}{4\pi r}I(\omega(t-r/c))\hat{z} = -\frac{\mu_0}{4\pi r}I_0 e^{(\omega(t-r/c)+\pi/2)}\hat{z}}$$

Fields

- Express Fields in terms of Source current

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t} = -\frac{\mu_0 p_0 \omega^2}{4\pi}\frac{\sin\theta}{r}\cos\left(\omega\left(t - r/c\right)\right)\hat{\theta} = \frac{\mu_0 \omega}{2\pi L}\frac{\sin\theta}{r} I\left(t - r/c\right)\hat{\theta}$$

$$\vec{B} = \vec{\nabla}\times\vec{A} = -\frac{\mu_0 p_0 \omega^2}{4\pi c}\frac{\sin\theta}{r}\cos\left(\omega\left(t - r/c\right)\right)\hat{\phi} = \frac{\mu_0 \omega}{2\pi L c}\frac{\sin\theta}{r} I\left(t - r/c\right)\hat{\phi}$$

Poynting vector

- Find average Poynting vecotr, power is found by integrating <s> over a sphere.

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{\mu_0}{c}\left\{\frac{p_0\omega^2}{4\pi}\frac{\sin\theta}{r}\cos\left(\omega\left(t-r/c\right)\right)\right\}^2\hat{r}$$

$$\left\langle\vec{S}\right\rangle = \frac{\mu_0}{c}\frac{p_0^2\omega^4}{32\pi^2}\frac{\sin^2\theta}{r^2}\hat{r}$$

$$\left\langle P\right\rangle = \oint\left\langle\vec{S}\right\rangle\cdot d\vec{a} = \frac{\mu_0}{c}\frac{p_0^2\omega^4}{32\pi^2}\int\frac{\sin^2\theta}{r^2}r^2\sin\theta\cos\phi = \frac{\mu_0}{c}\frac{p_0^2\omega^4}{12\pi}$$

$$\boxed{\left\langle P\right\rangle = \frac{\mu_0}{c}\frac{\omega^2\left(2L\right)^2}{12\pi}I_0^2}$$

Radiation Resistance

- Find average Poynting vector, power is found by integrating <s> over a sphere.

$$\langle P\rangle = \frac{1}{2}\mathrm{Re}(VI^*) = \frac{1}{2}\mathrm{R}_r\,|I|^2$$

$$R_r = 2\frac{\mu_0}{c}\frac{\omega^2(2L)^2}{12\pi} = \frac{\mu_0}{c}\frac{\omega^2(2L)^2}{6\pi}$$

$$R_r = \frac{\omega^2}{c^2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{(2L)^2}{6\pi}$$

$$\boxed{R_r = Z_0\left(\frac{\omega}{c}\right)^2\frac{(2L)^2}{6\pi}}$$

Ohmic Resistance

- Power loss from thermal motion.

$$R_{ohmic} \equiv R_S \left(\frac{L}{\pi a} \right)$$

$$R_S \equiv \sqrt{\frac{\pi f \mu}{\sigma}}$$

$$R_{ohmic} = \sqrt{\frac{\pi f \mu}{\sigma}} \left(\frac{L}{\pi a} \right) = \sqrt{\frac{Z_0}{2\sigma} \frac{\omega}{c}} \left(\frac{L}{\pi a} \right)$$

$2L =$ antenna length

$a =$ antenna radius

$f =$ operating frequency

$\sigma =$ conductivity

Radiation Efficiency

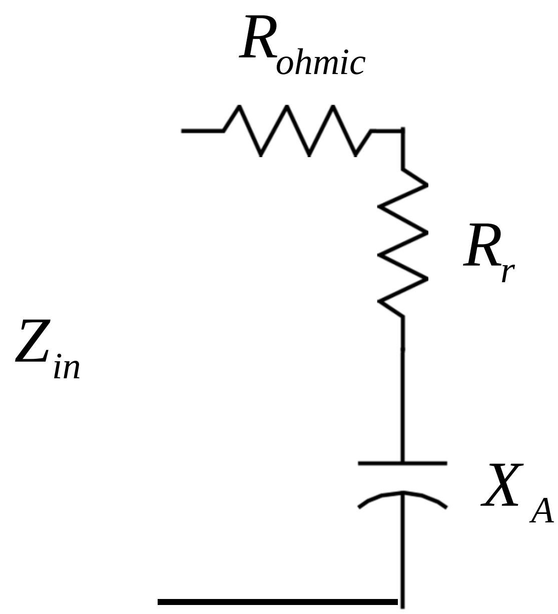
- Characterizing quality.

$$e_r \equiv \frac{R_r}{R_r + R_{ohmic}}$$

Input Impedance – Free Space

- Want to know power at field point as a function of current in the antenna.

$$Z_{in} = R_{ohmic} + R_r - jX_A$$



$$R_r = Z_0 \left(\frac{\omega}{c}\right)^2 \frac{(2L)^2}{6\pi}$$

$$R_{ohmic} = \sqrt{\frac{Z_0}{2\sigma}\frac{\omega}{c}}\left(\frac{L}{\pi a}\right)$$

$$X_A = -\frac{Z_0}{4\pi \frac{\omega}{c} L}\ln\left(\frac{2L}{a}\right)$$

Input Impedance – In Plasma

- Far above the plasma frequency, the power is like free space, but near the plasma frequency dispersion is important to consider.

$$\omega^2 = c^2k^2 + \omega_p^2$$

- The plasma frequency is the characteristic frequency of collective electron oscillations.

$$\omega_p^2 = \frac{n_e e^2}{\varepsilon_0 m_e}$$

- The index of refraction near to, but above the plasma frequency: (below the plasma frequency ordinary light does not propagate)

$$n = \frac{ck}{\omega} = \sqrt{1 + \left(\frac{\omega_p}{\omega}\right)^2}$$

$$Z = \frac{Z_0}{n}$$

- In the ionosphere it is more complicated because of magnetic field and resonances with ion and electron gyrofrequencies.

Cubesat Digisonde Antenna

- Antenna Specification

  $2L$ = antenna length=10m

  a = antenna radius=1 mm

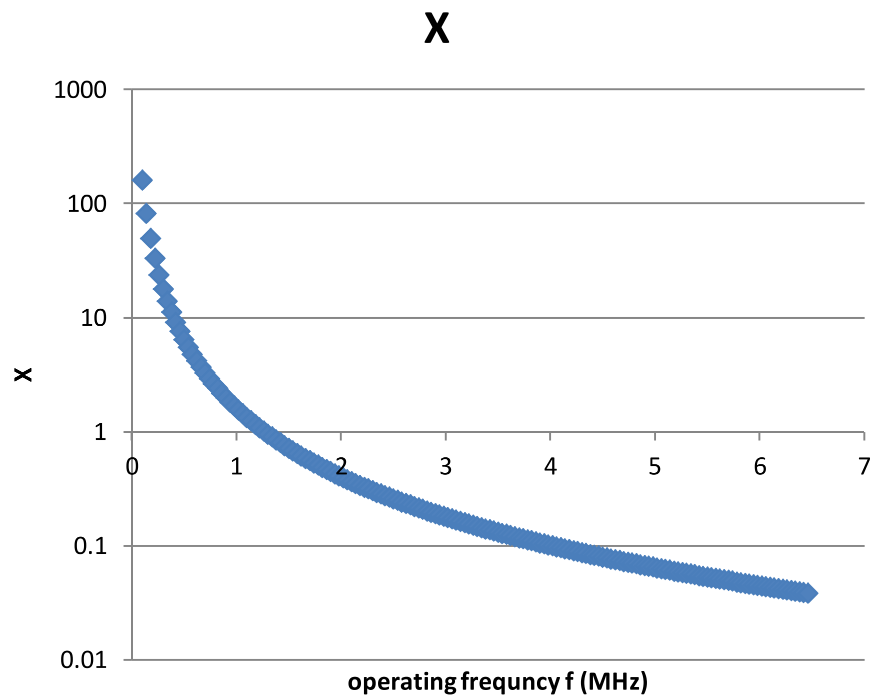
  f = operating frequency=0.1 to 6.46 MHz

  σ=conductivity=2e6 S/m

- Environment
  - 80°N 110°W 800 km
  - Ne=20,000/cm^3 and B=0.41G (IRI and IGRF)
  - fpe=1.27MHz
  - fce=1.15 MHz

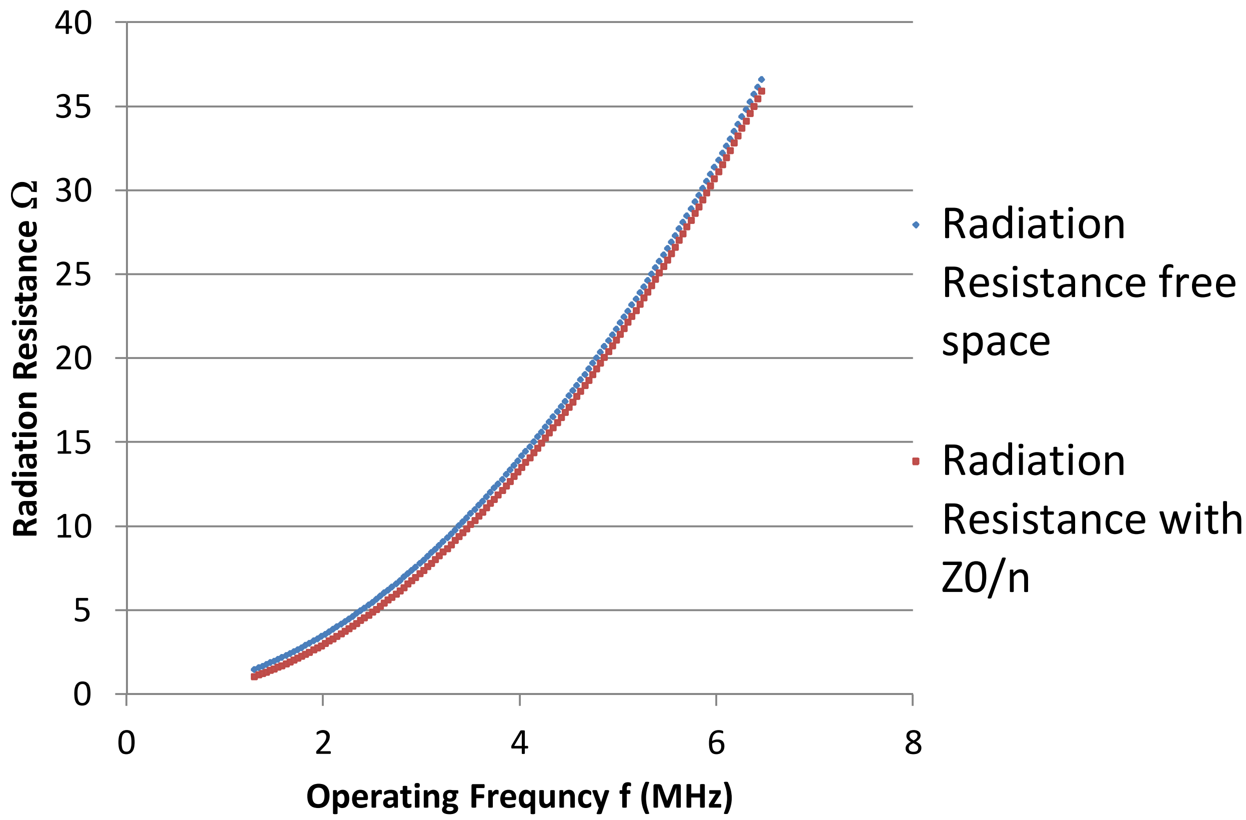
Cubesat Digisonde Antenna

- For this environment, at frequencies below 1.3 MHz there is no transmission.



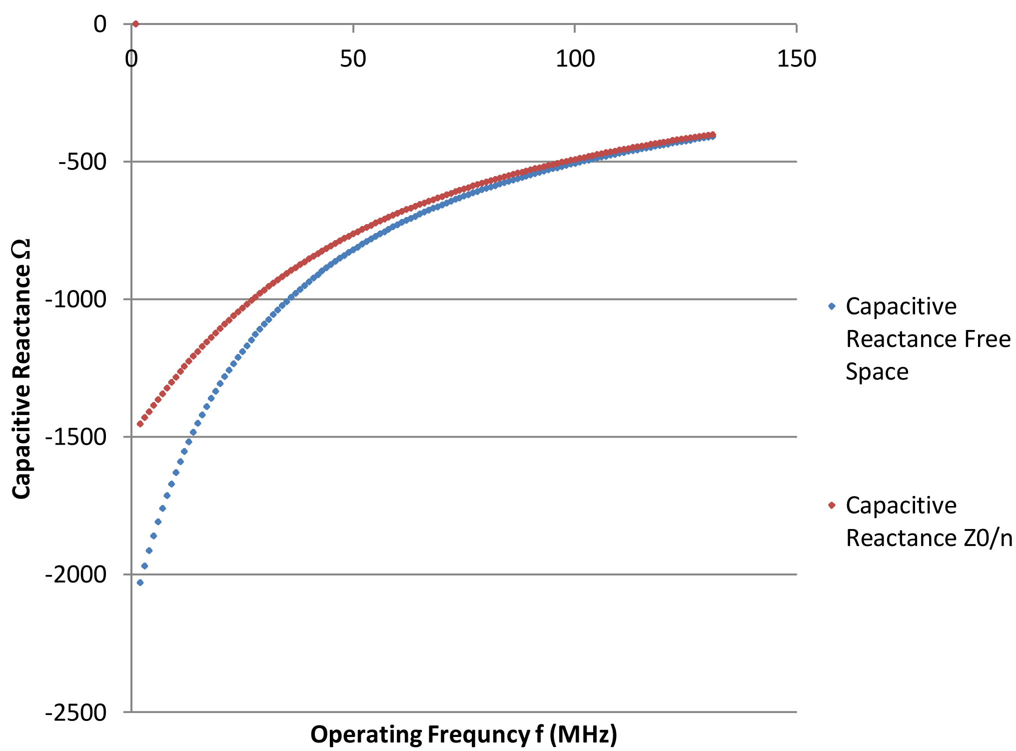
Cubesat Digisonde Antenna

- Free Space Radiation Resistance 1-40 Ω



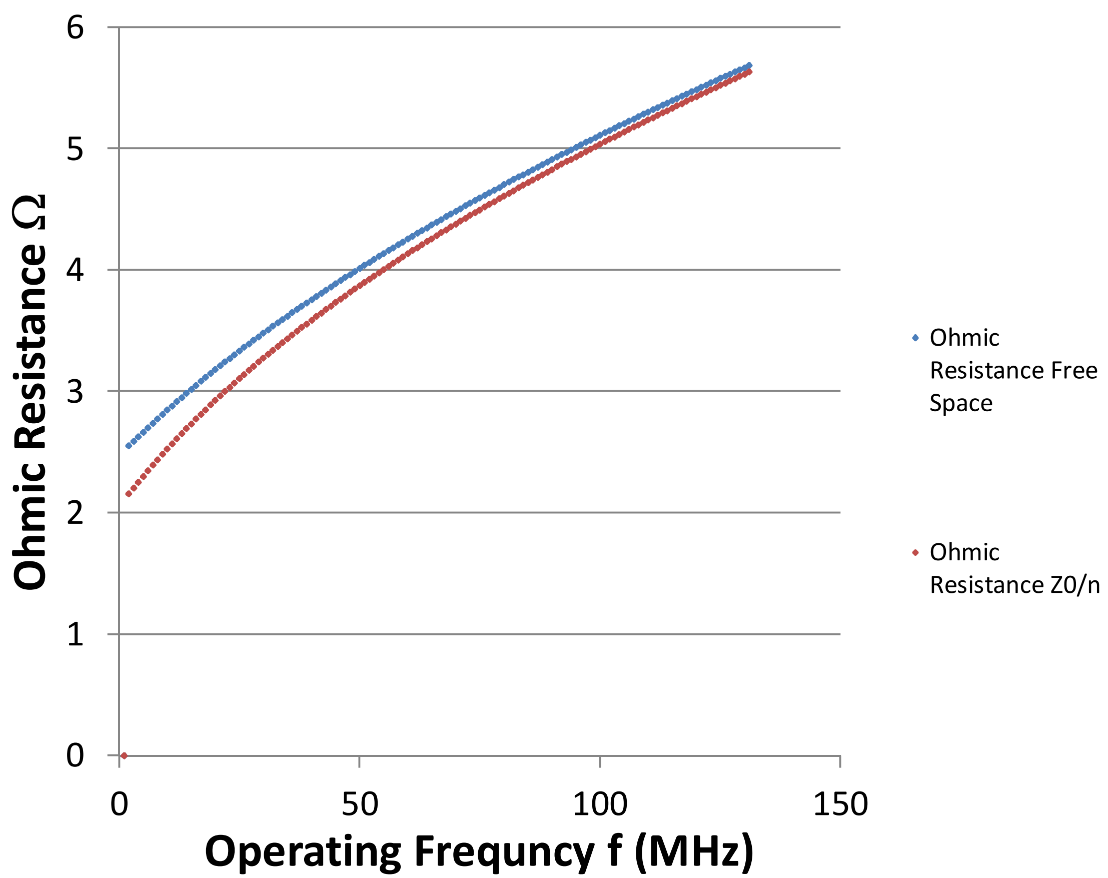
Cubesat Digisonde Antenna

- Capacitive Reactance.



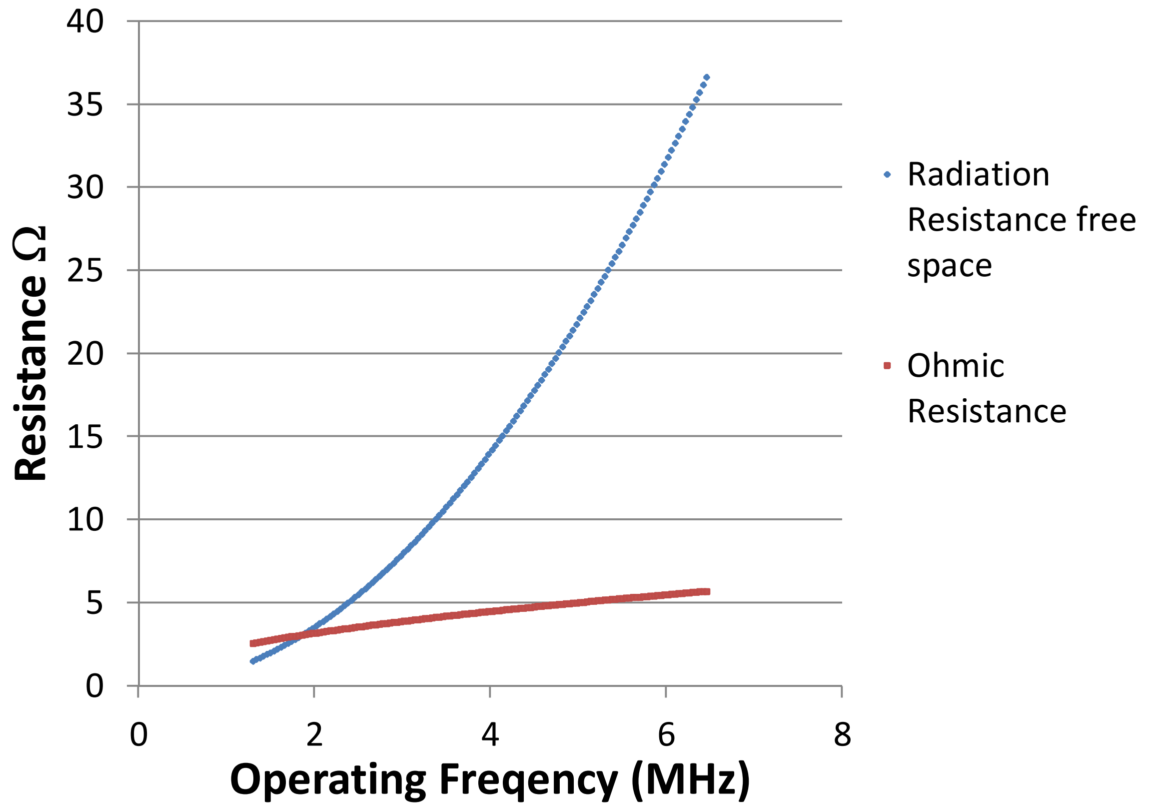
Cubesat Digisonde Antenna

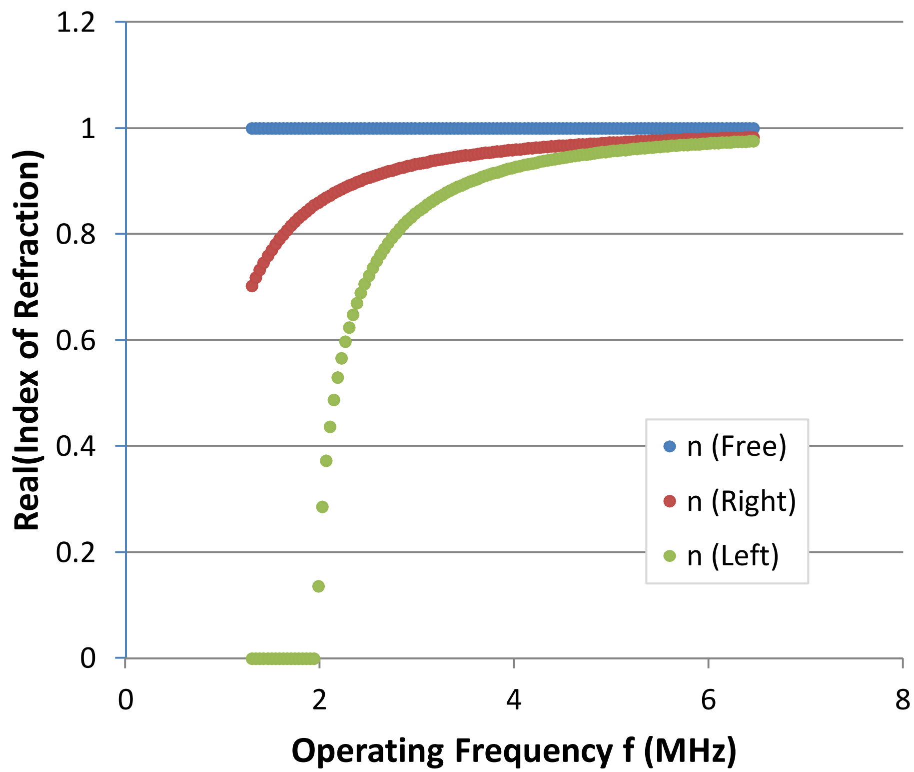
- Ohmic Resistance.



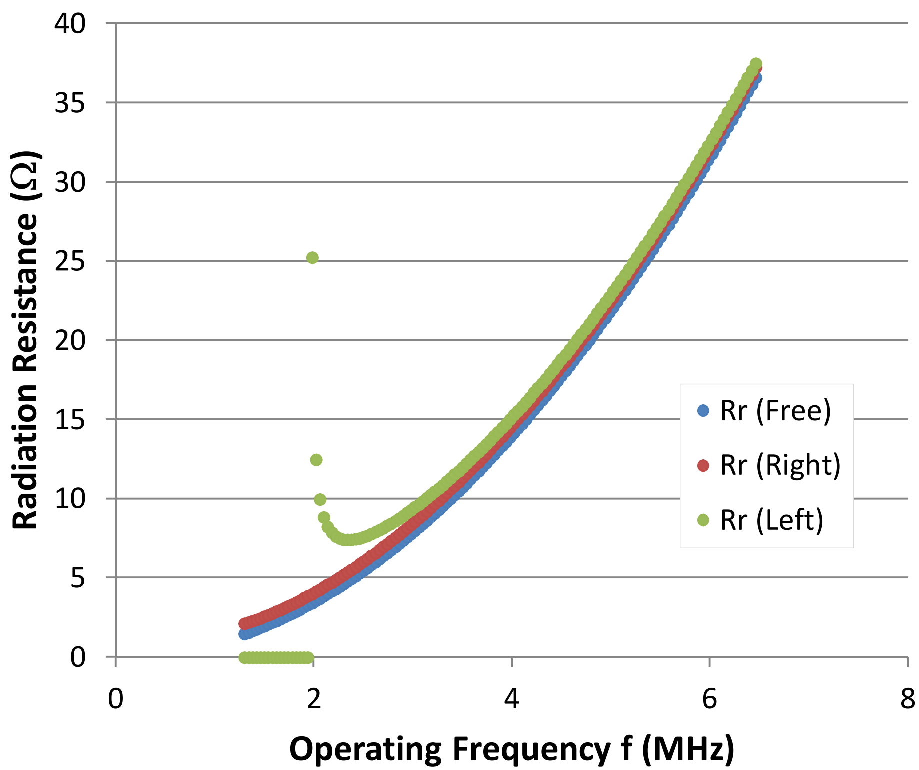
Cubesat Digisonde Antenna

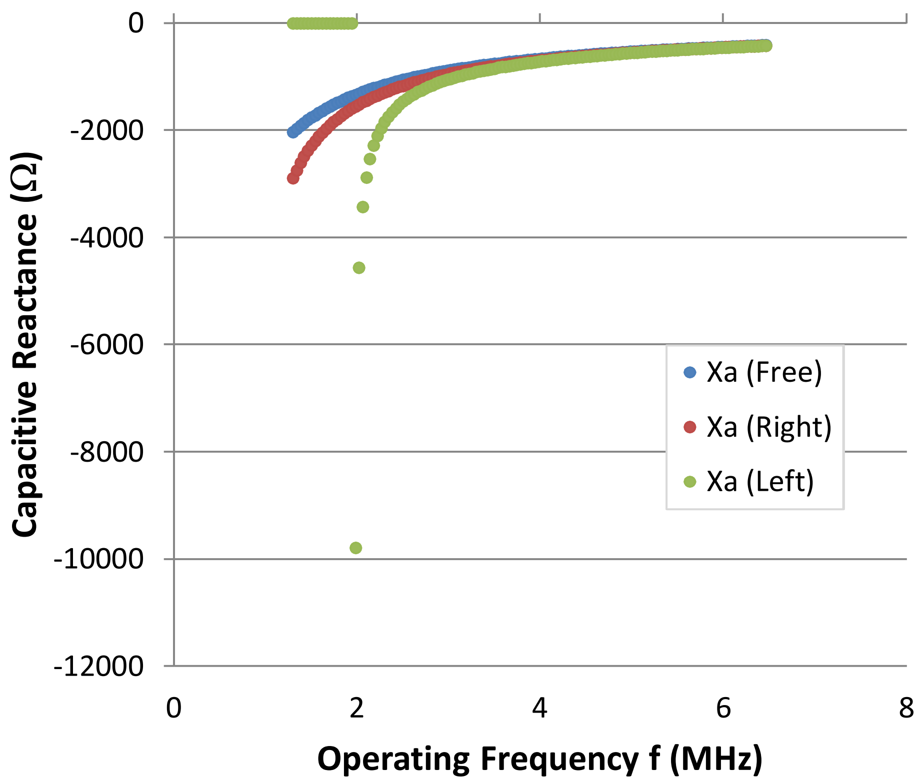
- Compare Radiation and Ohmic Resistance.




1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
Real(Index of Refraction)
Operating Frequency f (MHz)
n (Free)
n (Right)
n (Left)

Radiation Resistance




0
-2000
-4000
-6000
-8000
-10000
-12000
Capacitive Reactance (Ω)
0
2
4
6
8
Operating Frequency f (MHz)
Xa (Free)
Xa (Right)
Xa (Left)

Ohmic Resistance