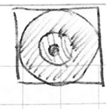
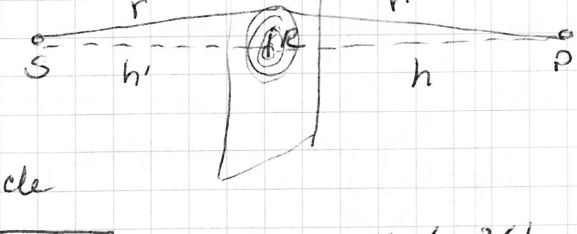
Fresnel Diffraction: Fresnel Zones



Point source + an aperture



construct zones/circles such that

$r + r' = \frac{1}{2}\lambda + h + h'$, R = radius of circle

use Pythagoras Theorem $\sqrt{h^2+R^2} + \sqrt{h'^2+R^2} = r + r' = h + h' + \frac{1}{2}R^2\left(\frac{1}{h'}+\frac{1}{h}\right) + \ldots$

do a binomial expansion $R_1 = \sqrt{\lambda L}$ $R_2 = \sqrt{2\lambda L}$ $L = \left(\frac{1}{h}+\frac{1}{h'}\right)^{-1}$

the radii of the Fresnel zones R_1, R_2

Can calculate the annular area

$= \frac{1}{2}\frac{2\pi h}{\lambda} h \cos\theta \Delta\theta$

$\pi R_{n+1}^2 - \pi R_n^2 = \pi R_1^2$ each zone has equal area

difference in phases arises from path length difference of $\lambda/2$

add total disturbance from each area

U_1 = Fresnel zone #1

$U_P = U_1 - U_2 + U_3 \ \ldots$

What happens is, as you move away from the center $\cos(n_0 r') + \cos(n r)$

obliquity factor depends on the sin of angle + the whole system changes.

$U_P = \frac{1}{2}|U_1| + \left(\frac{1}{2}|U_1| - |U_2| + \frac{1}{2}|U_3|\right) + \left(\frac{1}{2}|U_3| - |U_4| + \frac{1}{2}|U_5|\right) + \ldots$

if you take the average value between alternating zones   fresnel lens

block alternating zones + you can get increased intensity used to

all that contribute with equal phase + amplitude   Square of optical disturbance

Fresnel Diffraction ⇒ Fresnel zones

non-intuitive completely open aperature

block-off even order zones and (p. 129) (zones have equal area) increases intensity at observation pt

used for focusing pt srcs of light esp. in radio regime to avoid needing thick lenses of 1000's of wavelengths for reasonable #λ's of material to get focusing

focussed using fresnel zone plate

h src pt
h' obs pt.

$\frac{1}{h} + \frac{1}{h'} = \frac{1}{L}$ (5.36) resembles thin lens formula

L ← "effective" focal length

where $R^2 = \lambda L$

$R_n^2 = n\lambda L$ $L = \frac{R_n^2}{n\lambda} = \frac{R^2}{\lambda} = f_{eff}$

Stong dependence on λ

compared to thin lens which has less dependence on λ due to n index of refraction

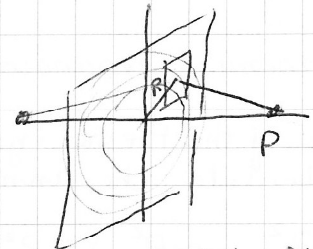
Fresnel Diffraction from Rectangular Aperture
use Kirchoff integral formula

p. 130 eq. 5.41 $\quad r+r' = h+h' + \frac{1}{2L}(x^2+y^2)$ from binomial expansion

$$U_p = C\int_{x_1}^{x_2}\int_{y_1}^{y_2} e^{ik(x^2+y^2)/2L}\,dx\,dy = C\int_{x_1}^{x_2} e^{ikx^2/2L}dx \int_{y_1}^{y_2} e^{iky^2/2L}dy$$

$L = \frac{R^2}{\lambda} \qquad u = x\sqrt{\frac{k}{\pi L}} \qquad v = y\sqrt{\frac{k}{\pi L}}$

$$U_p = U_1 \int_{u_1}^{u_2} e^{i\pi u^2/2}du \int_{v_1}^{v_2} e^{i\pi v^2/2}dv$$



can't do analytically over small regions

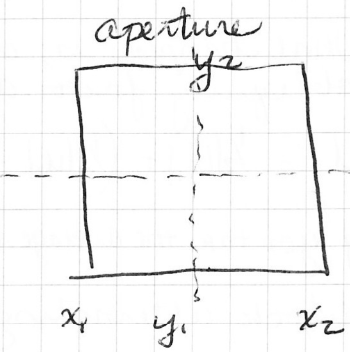
$C(s) = \int_0^s \cos(\pi u^2/2)du \qquad S(s) = \int_0^s \sin(\pi u^2/2)du$ and $\int_0^s e^{i\pi w^2/2}dw = C(s)+iS(s)$

using $e^{i\theta} = \cos\theta + i\sin\theta$

doing these integrals analytically is not possible - do numerically

table 5.2 fresnel integrals p.131

1) numerically do integral
2) get U_p at different positions
3) take square to get optical disturbance
   Can get intensity distribution

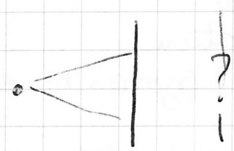


Cornu plotted $C(s)$ and $iS(s)$

painful
numerical values

easier to figure out what the pattern for simple geometry aperture

1D straight edge



rewrite the integral 5.44
$\Rightarrow$ each part has a $C+iS$

$$U_p = U_1 \underbrace{\int_{u_1}^{u_2} e^{i\pi u^2/2}du}\ \underbrace{\int_{v_1}^{v_1} e^{i\pi v^2/2}dv}$$

position via $v = y\sqrt{\frac{k}{\pi L}}$

$$U_p = U_1[C(s)+iS(s)][C'(s)+iS'(s)]$$

take a completely unobstructed source

unobstructed illumination

7.3
8.2

General form 5.47

$$U_p = \frac{U_0}{(1+i)^2}\left[C(u) + i(S(u))\right]_{u_1}^{u_2}\left[C(v) + iS(v)\right]_{v_1}^{v_2}$$

one dimension is infinite u dimension $u_1 = -\infty$ $u_2 = +\infty$

$$\left[C(s) + iS(s)\right]\Big|_{-\infty}^{+\infty} = (1+i)$$

$$U_p = \frac{U_0}{1+i}\left[C(u) + iS(u)\right]\Big|_{v_1}^{v_2}$$

$Z = C + iS$ is a complex vector on the cornu spiral plot

$C + iS$ is the amplitude

$$U_p = \frac{U_p}{1+i}\left[(C(v) + iS(v))\right]_{-\infty}^{v_2}$$ this is straight edge

$$U_p = \frac{U_0}{(1+i)}\left[C(v_2) + iS(v_2) + \tfrac{1}{2} + \tfrac{1}{2}i\right]$$

to get field point go to plot
$v_2 = 0$
comes out to be equal to $1/2$ - each component $v_2 = 0$

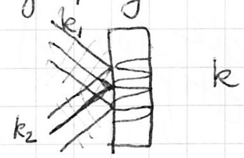
$$U_p = \left(\frac{U_0}{1+i}\right)\left[\tfrac{1}{2} + \tfrac{1}{2}i\right] = \tfrac{1}{2}U_0$$

U_0 is the unobstructed amplitude
can use cornu spiral to find other pts.

Fresnel Diffraction
- use Kirchoff - Fresnel formula, but integrals are difficult
- plot real + imaginary parts cornu spirals.
- difficult because of quadratic terms in the integral arising from the curvature of the wavefront.

fresnel integrals - you need the cornu spiral

Optics class - May 13th - Tuesday
photopolymer 10's of microns thick weakly absorbing change refractive index + absorption
holography is recording
white light comes
holography is the recording of a complex interferogram

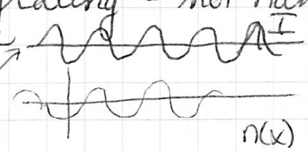


period of interference pattern

$\vec{k}_2 - \vec{k}_1 = \Lambda$ = period of interference pattern

Suppose interference pattern changes abs./ref index of pattern
periodic modulation
simple grating - not have the fidelity
saturate suppose modulation is 1

$I(x)$



Suppose it is a permanent change in real-time holography
suppose you could fix it
enough resolution (silver grains) very small silver halite grain sizes
develop then fix have the negative form a permanent change
amplitude grating send light thru it and it will deflect
exactly in same direction as other beam.

Simplest kind of hologram - no information inscribed - inscribed a grating
of two interfering plane waves.

Immediately realize Dennis Gabor - theoretical background
just demonstrated it - paper in Nature - not yet lasers

Collimated beam - light is scattered

Put through a pin hole - focus + recollimate
removes the

polymer sheet

Spatial filtering of remove high frequency components

light diffracts thru a pinhole - throw away a lot of light - so use strong laser

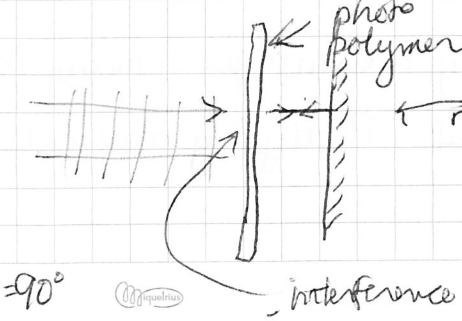
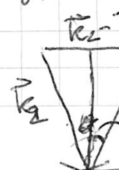


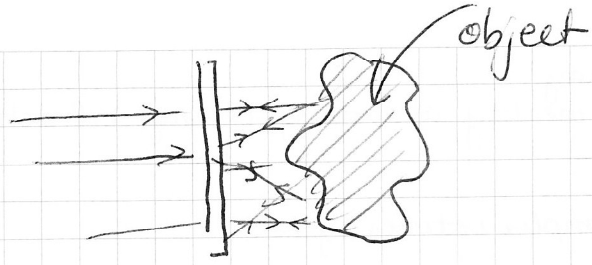
photo active material
record refractive index changes

$\Lambda = \frac{\lambda}{2n}$ at $\theta = 90°$

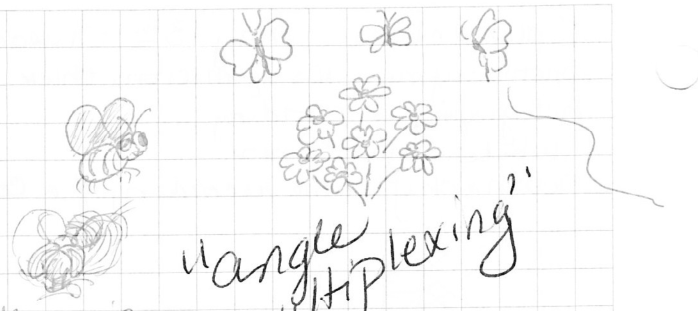


$\Lambda = \frac{\lambda}{2n\sin\theta}$

½ angle = θ



small Δn fix
permanent
(starts to deteriorate over time)



"angle multiplexing"

don't change all the refractive index all at once
get contrast
regime where changes in photographic film or polymer is linear
that's what this shows. fuck everything as long as you don't get into

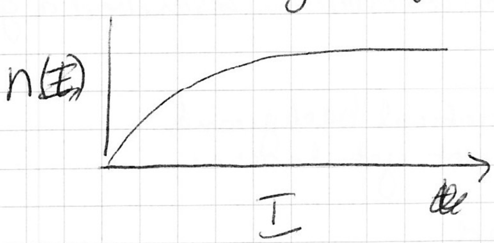
changes in absorption materials are almost transparent

illuminate it by the light
greatest colors you see Ar laser as tilt dif colors diffract
at different angles

tilt grating to get dif colors
what light does is "read" the complex grating

IBM just announced going to market holographic optical memory

Real-time holography - doesn't get permanently fixed non-linear optics



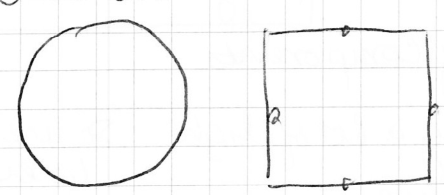
If the intensity is low - it never saturates - flattens interference pattern

decays with a time constant

so diffracted beam changes

Beat drum heads

vibration modes of a string



Boundary Conditions

a few miliseconds.

used in vibration analysis

holographic memory advantage - many MByte - good fidelity

Wall Street Journal Pretty Big application

Fourier Transforms

Fowles p. 71 Sec. 3.6
- Spectral Resolution of Finite Wave Train
- Coherence and Line Width

Properties of Light
- no light is strictly monochromatic
- spread of frequencies about some $\langle \nu \rangle$ = line width
- there is a relationship between line width and coherence length

$\boxed{\text{Fourier Transform Theorem}}$

A function $f(t)$ can be expressed as an integral over the variable ω in the following way:

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(\omega) e^{-i\omega t} d\omega \qquad g(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(t) e^{i\omega t} dt$$

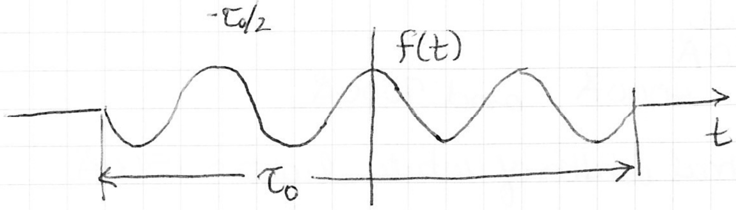
The functions $f(t)$ and $g(\omega)$ are Fourier Transforms

$g(\omega)$ = frequency resolution of $f(t)$
represents $f(t)$ in the frequency domain

$\boxed{\text{Example: Finite Train (Single Wave)}}$

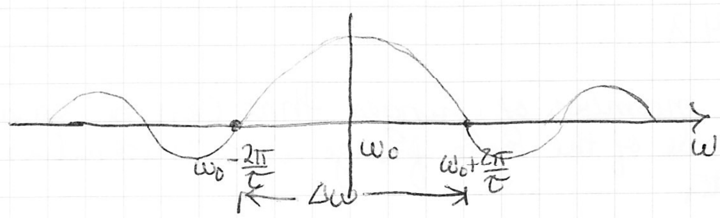
$f(t) = e^{-i\omega_0 t} \qquad -\tau_0/2 < t < \tau_0/2$
$f(t) = 0 \qquad$ otherwise

$$g(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\tau_0/2}^{+\tau_0/2} e^{i(\omega-\omega_0)t} dt = \sqrt{\frac{2}{\pi}} \frac{\sin[(\omega-\omega_0)\tau_0/2]}{\omega-\omega_0}$$

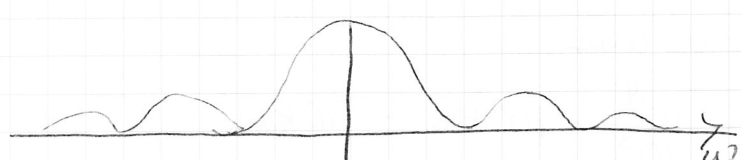


$f(t)$

Intersting



$g(\omega)$



$$G(\omega) = |g(\omega)|^2 = \frac{2\sin^2[(\omega-\omega_0)\tau_0/2]}{\pi(\omega-\omega_0)^2}$$

Spectral Distribution:

$\Delta\omega = \frac{2\pi}{\tau_0} \qquad \Delta\nu = \frac{1}{\tau_0}$

Coherence Length (from Fourier Transform argument)

Example: Sequence of Wave Trains

each lasts τ_0 duration - or can consider $\langle\tau_0\rangle$ for a number of random duration
random intervals

Reverse Reasoning ... Spectral Source has $\Delta\nu$

if given $\Delta\nu \Rightarrow \langle\tau_0\rangle = \frac{1}{\Delta\nu}$

$\Rightarrow$ the coherence length $l_c = c\langle\tau_0\rangle = \frac{c}{\Delta\nu}$

$\frac{\Delta\nu}{\nu} = \frac{|\Delta\lambda|}{\lambda}$ $\therefore$ $l_c = \frac{\lambda^2}{\Delta\lambda}$

Specific Example

discharge tubes $\Delta\lambda \sim 1Å$ $\lambda_0 \approx 5000Å$

$l_c = \frac{\lambda^2}{\Delta\lambda} \sim 5000\lambda \sim 2mm$

in an interference experiment the fringe visibility $\mathcal{V}$ would be vanishingly small for path differences much larger than this ~~wave~~ coherence length

Sensitivity of Eyes
max @ 5580Å
falls to zero 4000Å and 7000Å

White Light

to eye, spectral width of white light ~ 1500Å

$l_c \sim 3$ or 4λ

3 (or 4) this is the number of fringes that can be seen on either side of the zero fringe in a Michaelson interferometer

Gas Laser

$\Delta\nu \sim 10^3$ Hz or less

$l_c \sim \frac{\nu}{\Delta\nu} \sim \frac{10^{14}}{10^3} = 10^{11}$ Hz ~ 50km

① interference can be detected over long distances

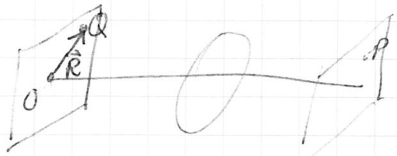
② fringes can be produced using two different sources but it is not steady
fluctuates on coherence time $\sim 10^{-3}$ s

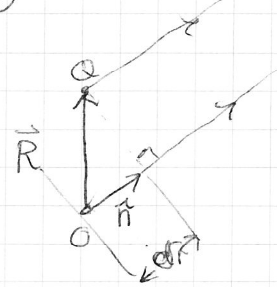
Fourier Transform

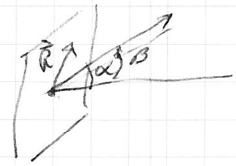
Applications to Diffraction p. 135 Fowles

aperature of arbitrary shape, varying transmission & phase retardation

all rays leave diffracting aperture α, β, γ are brought into common focus







$\vec{R} = \hat{i}x + \hat{j}y$ $\quad\hat{n}$ is unit vector

$\delta r = \vec{R}\cdot\hat{n} = x\alpha + y\beta = x\frac{X}{L} + y\frac{Y}{L}$

diffraction eq → $U(X,Y) = \iint e^{ik\delta r}\, dA = \iint e^{ik(xX+yY)}\, dx\,dy$ uniform aperture

$g(x,y)$ = aperture function

$U(X,Y) = \iint g(x,y)\, e^{i(Xx+yY)}\, dx\,dy \qquad \mu = \frac{kX}{L}$ and $\nu = \frac{kY}{L}$

$U(\mu,\nu) = \iint g(x,y)\, e^{i(\mu x+\nu y)}\, dx\,dy$

fourier transform pair

Consider example of a grating

$g(y) = g_0 + g_1\cos(\nu_0 y) + g_2(2\nu_0 y) + \dots$

$\nu_0 = \frac{2\pi}{h}$