that lateral coherence of $\boxed{l_c = \frac{1.22\lambda}{\theta}}$

used in astronomy to find sizes of stars)

Read book, problems

Visibility of fringes, interference of thin films

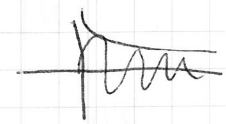
won't include fourier series + fourier transforms
next exam
getting 1st few terms
simple FT

FT of $f(t) = e^{-\alpha t} e^{i\omega_0 t}$ $e^{-\alpha t}$
should be simple to do

$$f(\omega) = \frac{1}{\sqrt{2\pi}} \int_0^\infty (e^{-\alpha t} e^{i\omega_0 t}) e^{-i\omega t}\, dt$$

damped harmonic
oscillator

$$f(\omega) = \left[\frac{e^{-i(\omega_0 - \omega)t - \alpha t)}}{i(\omega_0 - \omega) - \alpha} \right]_0^\infty$$



Power Spectrum

$$|f(\omega)|^2 = \frac{1}{(\omega_0 - \omega)^2 + \alpha^2} \leftarrow \text{Lorenzian}$$

mimicks
what happens to harmonic
oscillator.

ability to use the material you have learned
book + notes allowed

Tues

Fabry-Perot int. very precise wavelengths
MHz freq. diff - very precise 1 part per billion
follows book closely

$\delta = 2kd\cos\theta$ phase Δ due to path length diff

$r = |r| e^{i\delta_r/2}$
$\nwarrow$ phase Δ of 1 reflection

$$I = \frac{I_0 |t|^4}{|1 - r^2 e^{i\delta}|^2}$$

$R = r^* r = |r|^2$

$T = t^* t = |t|^2$

$\frac{\Delta}{2} = \Delta = 2N\pi = \frac{4\pi}{\lambda_0} n d\cos\theta + \delta_r$

every time part of fn goes to 1

$\delta/2 = n\pi$ $\quad I_T = \_\_\_\_$
looks like
resonances

how to measure
wavelength diff exactly

$\Delta N \qquad \Delta(N+1)$ resonances

pinhole scan

$$\boxed{2\pi = \frac{4\pi}{\lambda_0} n \Delta d}$$

distance between

$$\Delta d = \Delta_{N+1} - \Delta_N = \frac{\lambda_0}{2n}$$

move distance
between two plates go from
one distance
scan.

$\Delta/2 = \delta + \delta_r = N\pi$

$$\Delta_N = 2N\pi = \frac{4\pi}{\lambda_0} nd\cos\theta + \delta_r$$

circles
sect 15
cond for

free spectral range $\nu_{N+1} - \nu_N = \frac{c}{2nd}$

Fabry Perot fringes

Homework
4.1, 4.2, 4.7, 4.8, 4.9

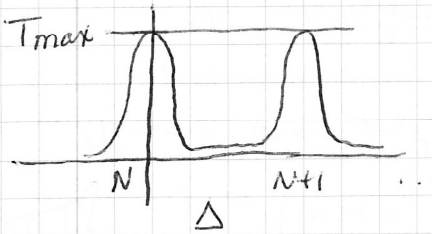
Fabry Perot etalon change – Scan interferometer – Δ relative spacing btwn 2 mirror plates

Central fringe $\cos\theta \sim 1$

$$\Delta = \frac{4\pi n d}{\lambda} + \delta(\lambda)$$

refractive index doesn't change rapidly over a few Å

Called Scanning the interferometer



separation = free spectral range [GHz]

$(\Delta\nu_{N+1} - \Delta\nu_N) = \frac{c}{2nd}$ $\Delta_{N+1} - \Delta_N = 2\pi$

frequency (4.16)

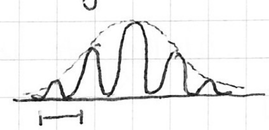
width
- vibrations Δ cavity length

$(\Delta\nu_{N+1} - \Delta\nu_N)$ = can be of order 100 MHz or less ($d \sim 1m$)

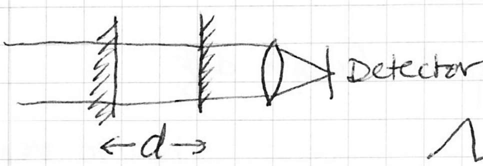
HeNe laser – buy a very short cavity length 7.5cm long mode spacing ~2GHz

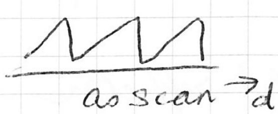
one laser downstairs that puts out green and blue light

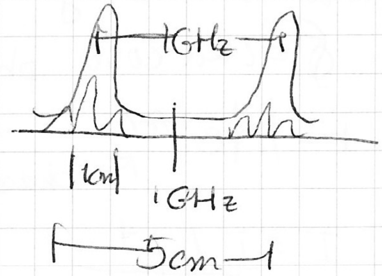
laser line profile



Fabry-Perot gives very high resolution spectroscopy







1cm = 200MHz

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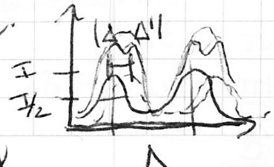
thin etalon with low finesse – removes other lines
position the peak that was there.

figure in book

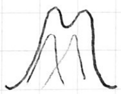
experimentally how it is used



4.6
p.94

Assumption is the two lines have equal intensity
Resolve the two

Taylor Criterion – Saddle Point   Rayleigh



$$I = 2I_0\left[1 + F\sin^2\left(\frac{\Delta - \Delta'}{4}\right)\right]^{-1}$$

Coef of finesse

$$F = \frac{4R}{(1-R)^2}$$ Airy Function

$$R = |r|^2 = rr^*$$

$$F\sin^2\left(\frac{\Delta-\Delta'}{4}\right) = 1$$

$$\Rightarrow |\Delta - \Delta'| = 4F^{-1/2}$$ small differences

Airy function

$$\frac{1}{1 + F\sin^2(\Delta/2)}$$

$$R.P. = \frac{\lambda}{|\Delta\lambda|} = \frac{\omega}{\delta\omega} = \frac{\nu}{\delta\nu} = N\mathcal{F} = N\pi\left(\frac{\sqrt{R}}{1-R}\right)$$

$$\delta\omega = 2\pi\delta\nu = \frac{2c}{d}F^{-1/2}$$

$$\Delta = \frac{4\pi}{\lambda_0}d(\nu) + \delta_\nu \qquad \Delta = \frac{4\pi d}{\lambda} \sim 2N\pi$$ condition for maxima

$$2\pi\nu = \frac{c}{\lambda}2\pi$$

$$\frac{\omega}{\Delta\omega} = \frac{2\pi}{\lambda}\frac{d}{2}F^{1/2} = \frac{\pi N F^{1/2}}{2} = \frac{\lambda}{\Delta\lambda}$$

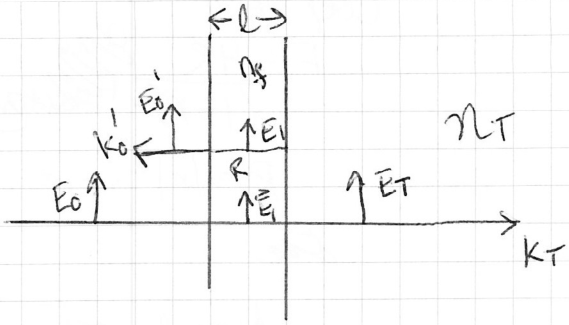
typically $N \sim 10^5$

Resolving Power

$$R.P. \sim 10^7 = \frac{\lambda}{\Delta\lambda} = \frac{\omega}{\Delta\omega}$$

light $\sim 10^{14}$ Hz with RP 10^7 a few MHz resolution

NOTE BOOK



tangential component $\left\{ \begin{matrix} H \\ E \end{matrix} \right\}$ is conserved

(homework problem for arbitrary angle)

one of the homework problems ask you to do this algebra.

matrix form 4.23

$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix} \frac{E_0'}{E_0} = \begin{bmatrix} \cos kl & -\frac{i}{n_1}\sin kl \\ -in_1 \sin kl & \cos kl \end{bmatrix} \begin{bmatrix} 1 \\ n_T \end{bmatrix} \frac{E_T}{E_0}$$

1ST interface

$E_0 + E_0' = E_1 + E_1'$

$H_0 - H_0' = H_1 - H_1'$

$nE_0 - n_0 E_0' = n_1 E_1 - n_1 E_1'$

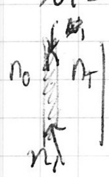
2ND interface

$E_1 e^{ikl} + E_1' e^{-ikl} = E_T$

$H_1 e^{ikl} - H_1' e^{-ikl} = H_T$

$n_1 E_1 e^{ikl} - n_1 E_1' e^{-ikl} = n_T E_T$

$M = M_1 M_2 M_3 \cdots M_N$



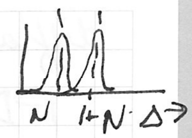
just one layer

chose thickness and refractive index
suppose make $kl = \frac{\pi}{2}$ quarter of wavelength $\lambda/4$

p.99 MgF to reduce losses in laser

Laser mirrors multilayer coatings

Fresnel-Kirchoff Diffraction Formula 5.11 p. 110

F.P. $RP = \frac{\lambda}{\Delta\lambda} = \frac{\omega}{\Delta\omega} = \frac{\nu}{\Delta\nu} = \frac{N\pi\sqrt{R}}{(1-R)}$  $\Delta = \frac{4\pi}{\lambda} nd(\cos\theta) + \delta_r \Rightarrow m = N = \frac{2d}{\lambda}$ 

Multilayers

$|r|^2 \leftarrow n_0 | n_1 | n_2 | n_3 \rightarrow |t|^2$ ($\lambda/4$) minimize the reflection

$n_1 = \sqrt{n_T}$ $r = 0$ making antireflection coatings

Example: Si Solar Cell (n is high 3.5 or 3.6) 30% is reflected want to harvest all the light

Solution: put on a thin film of dielectric refractive index of $\sqrt{3.5} \sim 1.8$

quarter of wave $\frac{1}{4}\left(\frac{550\,nm}{1.8}\right) \approx 60\,nm$ is about the thickness you want

wavelength in medium (λ_0/n)

reduces the reflectivility of the solar cell to about 1% or 2%

Diffraction - another manifestation of wave phenomena
bending of light as it goes through a narrow slit

circular wave front
Huygen's take each point on the wavefront as a source of a circular wave

lead to question:
if we take each pt as a source
why don't we see the pattern going back?
soln: Kirchoff - net disturbance pattern shown
assuming two optical waves U V

$\nabla^2 U = \frac{1}{u^2}\frac{\partial^2 U}{\partial t^2}$ $\nabla^2 V = \frac{1}{u^2}\frac{\partial^2 V}{\partial t^2}$

$U = \frac{U_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}}{r}$ $V = \frac{V_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}}{r}$

p 110

Kirchoff:
Accept Green's Theorem as fact, apply to an aperture (5.12) p. 110

$$U_p = \frac{-ik}{4\pi}\iint \frac{U_A e^{i(kr-\omega t)}}{r} \underbrace{[\cos(\vec{n},\vec{r}) + 1]}_{\text{obliquity factor}} dA$$

$$U_A = \frac{U_0 e^{ikr'}}{r'}$$

looking in the backward direction the obliquity factor is zero

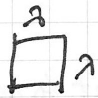
factor of $-i$ tells you the ~~reflected~~ scattered/diffracted field is phase shifted by 90°

another simplification: U is a scalar function E/M waves are vectors
in late 1930's/1940's vectors become more complicated
most diffraction phenomena you can explain using scalar fields
metal apertures) isotropic properties) — get dif results than Kirchoff's formula.

diffraction effects are negligable — a few hundred microns

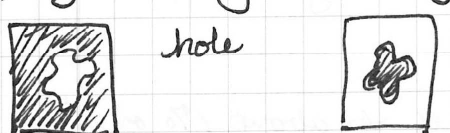
1-D means 1-dimension is much longer.

2-D



aperature with area elements have a different r (distance between aperature & field point/screen @ P)

Apply to any arbitrary aperture



hole

remove the outerpart make aperature, window

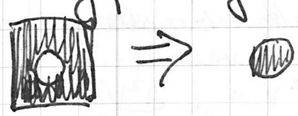
net disturbance due to both. $U_{1P} + U_{2P} = 0$

then put cut-out piece & remove wall.

hole in the wall $\boxed{U_{1P} = -U_{2P}}$ actually the diffraction pattern due to plug is opposite field amplitude

complementary.

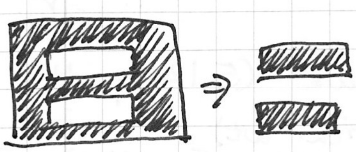
very powerful result



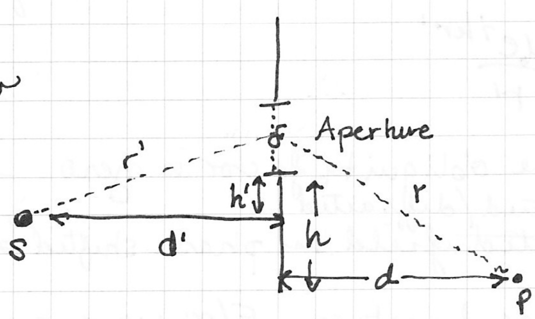
intensity pattern due to hole & disk are the same

babinet's principle Complementary apertures

Similarly — thin wire is the same as a thin slit



valid for both fresnel & fraunhofer diffraction



distances on order of 10's of cm $\gg \lambda$

$\delta \ll d$

use Pythagoras theorem

$$\Delta = \sqrt{d'^2 + (h'+\delta)^2} + \sqrt{d^2 + (h+\delta)^2} - \sqrt{d'^2 + h'^2} - \sqrt{d^2 + h^2}$$

assume that these distances — expand in terms of delta $\delta \ll d$

$$\Delta = \left(\frac{h'}{d'} + \frac{h}{d}\right)\delta + \frac{1}{2}\left(\frac{1}{d'} + \frac{1}{d}\right)\delta^2 + \cdots$$

Fraunhofer Diffraction

if $\frac{1}{2}\left(\frac{1}{d'}+\frac{1}{d}\right)\delta^2 \ll \lambda$ then the contribution to phase is negligible

assumes no curvature of the wavefront
can use fourier tranform.

Fresnel-like Diffraction & intermediate zone - compute numerically

---

Diffraction - use the result
apply Green's theorem to spherically symmetric waves
make approx at appratme
service = plane
Frainhaufer condition $\Rightarrow \frac{1}{2}(\frac{1}{d'}+\frac{1}{d})\delta^2 \ll \lambda$

Simple Case: Rectangular 1D aperture

plane

$u_A = \frac{u_0 e^{ikr'}}{r'}$

phase factor

obliquity factor $\quad U_p = -\frac{ik}{4\pi}\iint_A u_A \frac{e^{i(kr-\omega t)}\left[(\cos n, r)+1\right]}{r'}\, dA$

small effect compared to phase

r is large $\Rightarrow$ assume
forward
variation in r
phase oscillates rapidly

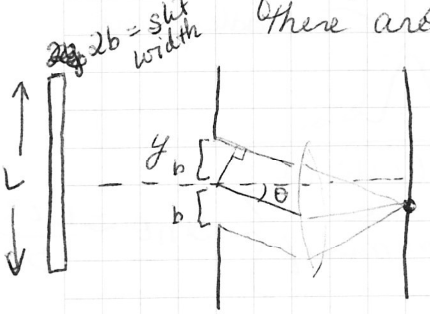
$k = 2\pi/\lambda$

Small variation in r on Angstroms causes big change in phase factor

if r changes by fraction of λ
there are large changes in phase

[you are OK]

2b = slit width



~~ysinθ~~

$r = r_0 + y\sin\theta$ $\quad e^{ikr_0}$ $\quad U_p = Ce^{ikr_0}\iint e^{iky\sin\theta}\, dA$

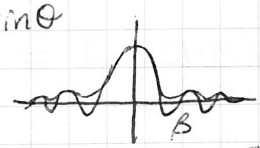
$dA = L\,dy$ $\quad$ L is the total long dimension of the slit $\quad dA = L\,dy$

$U_p = 2LCe^{ikr_0}\int e^{iky\sin\theta}\, dy$

optical disturbance amplitude

use the fact that $\frac{e^{i\theta}+e^{-i\theta}}{2} = \sin\theta$

$U_p = C'\left(\frac{\sin\beta}{\beta}\right)$ sinc function

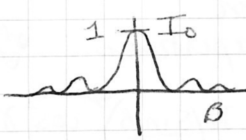


$\beta = \frac{1}{2}kb\sin\theta$

$n\pi$

Example:
slit = 100 μm in dimension = 10^{-2} cm
Calculate $\lambda = 500\text{nm} = 5\cdot10^{-5}$ cm

$\sin\theta = \frac{\lambda}{10^{-2}} = \frac{5\cdot10^{-5}}{10^{-2}} = 5\cdot10^{3}$ radians $\sim\theta$



$I \propto |U_p|^2 = I_0\left(\frac{\sin\beta}{\beta}\right)^2$

$\beta = \frac{1}{2}kb\sin\theta = n\pi$

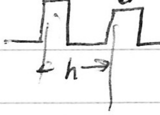
take geometry where $\frac{y'}{x} = 5\cdot10^{-3}$ $\quad y_1 = 5\cdot10^{-3}x$

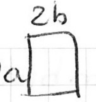
$\Rightarrow \frac{1}{2}\frac{2\pi}{\lambda}b\sin\theta = n\pi$

what about condition for maxima take derivative of Up to get transcendental equation
find out where they will occur (hmwk)

due to diffraction pattern of ... slit $\quad \Rightarrow b\sin\theta = n\lambda$

to a certain approx: $b\sin\theta \sim (n+\frac{1}{2})\lambda$ not exact $\quad$ Condition for minima

essentially for Fraunhaufer 
take fourier transform



$\beta = \frac{1}{2} k b \sin\theta$ $\quad 2a \quad 2b$ $\quad U_p = C\int e^{i(k_x x + k_y y)}$

$\alpha = \frac{1}{2} k a \sin\theta$

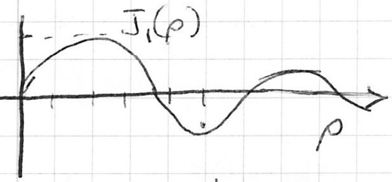
remember when you use cartesian coord → get sines + cosines
this for rectangular apertures

$U_p = \iint e^{ikr} dA$ circular aperture has cylindrical symmetry

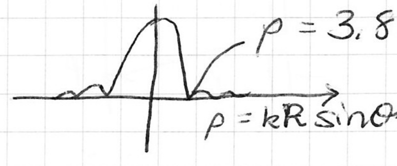
R = radius of aperture in book $\quad U_p = ~~\iint~~ \; C e^{ikr_0}$

$I = I_0\left[\frac{2J_1(\rho)}{\rho}\right]^2$

$\int_{-1}^{+1} e^{i\rho u}\sqrt{1-u^2}\,du = \pi \frac{J_1(\rho)}{\rho}$ ~~Bess~~ Bessel Functions



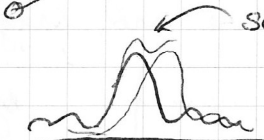
whole factor becomes close to 1



essentially here $\sin\theta$ term $\quad \rho = kR\sin\theta$

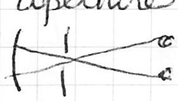
from the behavior of the zeros of the Bessel ftn, you can find where the minimums are
Bright central maximum got most of the intensity
unless you use a very strong laser see very few rings

$\sin\theta = \frac{3.832}{kR} \sim \frac{1.22\lambda}{D}$



Suppose you are looking at 2 headlights

iris of eye = circular aperture

D = diameter of iris 

Rayleigh Criteria
θ sep such that {max of one falls on min of other} $\quad \sin\theta = \frac{1.22\lambda}{D}$

let $D \sim 2\cdot10^{-3}$m
$\lambda \sim 5\cdot10^{-7}$m

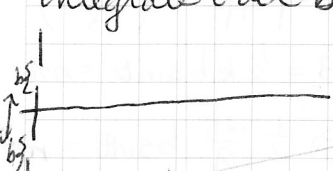
$\sin\theta = \frac{(1.22)(5\cdot10^{-7}m)}{(2\cdot10^{-3}m)} = 2.5\cdot10^{-4}$ radians

if headlights of car are separated by 1m then our distance to car is

$\theta = 2.5\cdot10^{-4} = \frac{1\,\text{meter}}{d}$

$d = \frac{1m}{2.5\cdot10^{-4}} \sim 4000m$

Double Slit diffraction
integrate over both apertures



envelop curve $\quad$ youngs max when

$I = I_0\left(\frac{\sin\beta}{\beta}\right)^2 \cos^2\gamma$

$h\sin\theta = n\lambda$ cond for maxima
$b\sin\theta = m\lambda$ what order

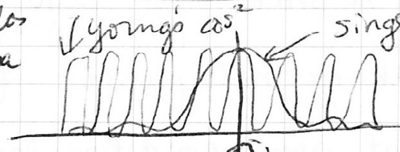
this is what we got for youngs experiment

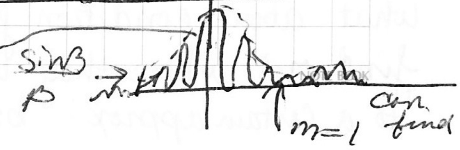
$\frac{b}{h} = \frac{m}{n}$

$\gamma = \frac{1}{2} k h \sin\theta$
$\beta = \frac{1}{2} k b \sin\theta$

$n = \frac{h}{b}$ missing order of youngs pattern

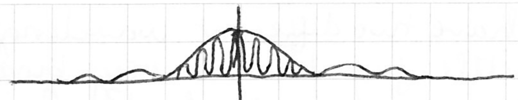
as $b \to 0 \quad \frac{\sin\beta}{\beta}$





Tuesday May 5

Double Slit and diffraction



Condition for (inside) max
$h(\sin\theta) = n\lambda$
min:
$b\sin\theta = m\lambda$

"6" is the missingorder
missing order found from ratios of slits

take intensity patternfor single slit (to get maxima's)
take derivative } $\tan\beta = \beta$

5.1, 5.2, 5.3, 5.5*, 5.13, 5.14 homework problems

p.122 $I = c\int e^{iky\sin\theta}\,dy$

N periodically separated slits (= separation h)
same size (b = slit width)
in the other dimension slits are long

$$= \int_0^b e^{iky\sin\theta}\,dy + \int_h^{h+b} e^{iky\sin\theta}\,dy + \dots$$

$$= \left\{\frac{e^{ikb\sin\theta}-1}{ik\sin\theta}\right\}\left[1 + e^{ikh\sin\theta} + e^{2ikh\sin\theta} + \dots e^{ik(N-1)\sin\theta}\right]$$

$$= \left\{\frac{e^{ikb\sin\theta}-1}{ik\sin\theta}\right\}\left[\frac{1-e^{ikNh\sin\theta}}{1-e^{ikh\sin\theta}}\right]$$

Convert to sin: $\frac{e^{i\theta}+e^{-i\theta}}{2i} = \sin\theta$

$$= be^{i\beta}e^{i(N-1)\gamma}\left(\frac{\sin\beta}{\beta}\right)\left(\frac{\sin N\gamma}{\sin\gamma}\right)$$

$\beta = \frac{1}{2}kb\sin\theta \quad \gamma = \frac{1}{2}kh\sin\theta$

$$I = I_0\underbrace{\left(\frac{\sin\beta}{\beta}\right)^2}_{\text{diffraction term}}\underbrace{\left(\frac{\sin N\gamma}{N\sin\gamma}\right)^2}_{\text{interference term}}$$

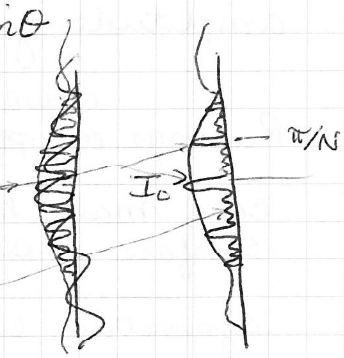
$\gamma = \frac{1}{2}kh\sin\theta \quad \beta = \frac{1}{2}kb\sin\theta$

intensity at central maximum
get maxima when $\boxed{\gamma = n\pi}$

$\frac{1}{2}kh\sin\theta = n\pi$ | condition for principle maxima

$\boxed{h\sin\theta = n\lambda}$

$\gamma = \frac{\pi}{N}, \frac{2\pi}{N}, \frac{3\pi}{N},$



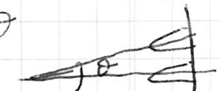
π/N

I_0

denominator on other hand - becomes identically equal to zero

Secondary maxima
$\frac{3\pi}{2N} \Rightarrow \gamma N = \frac{3}{2}\pi$ if N is large $\Rightarrow N\sin\gamma \to N\left(\frac{3\pi}{2N}\right) = \frac{3\pi}{2}$

all wavelength λ show up in central fringe
but the angular position of principle maxima depend on wavelength λ
(slits/mm) = $1/h$ Can determine λ by finding what angle θ the ~~diff~~ maxima is diffracted to



$\sin\theta = (\lambda/h)$ $\quad \lambda = h\sin\theta = \frac{\sin\theta}{\#\text{slits/mm}}$ gratings typically 1000's per mm

If you have two different wavelengths λ_1, λ_2 if they are separated by very little just have a very broad peak. What do we mean when two λ's are just resolved? Criteria: if maxima of one lies at minima of the other - assume the same intensity

Rayligh criteria: similar intensity

$\boxed{h \sin\theta = n\lambda}$ condition for primary maxima

as $N\uparrow \Rightarrow$ ~~narrower~~ sharper max peak and the ~~minima are very smaller~~ secondary max are smaller

$h\cos\theta \, d\theta = n\, d\lambda \Rightarrow \quad \Delta\theta = \dfrac{n\Delta\lambda}{h\cos\theta}$ ①

$\Delta\gamma = \dfrac{\pi}{N} = \Delta\left(\tfrac{1}{2}kh\sin\theta\right) = \tfrac{1}{2}kh\cos\theta\, d\theta$

just resolved?
these equations hold $\quad \Delta\theta = \dfrac{\lambda}{Nh\cos\theta}$ ②

$\Delta\theta =$

① + ② $\Rightarrow \dfrac{\lambda}{\Delta\lambda} = R.P. = nN \leftarrow$ number of slits used for the diffraction

$\uparrow$ order of diffraction

$RP\uparrow$ but $I\downarrow$
intensity goes down at higher orders

$\dfrac{5000\text{Å}}{\Delta\lambda} = 10{,}000 \quad \Delta\lambda = \dfrac{5\cdot 10^3}{10^4}$

grating spectrometers expand the beam to fill ___. usually can get less than 1Å

use as much of slits as you can to ___

F.P. is better than grating

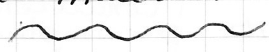
grooves per mm

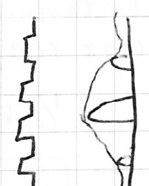
amplitude grating can get phase differences phase of light is periodically modulated.

advantage: greater light throughput - "all" light goes thru

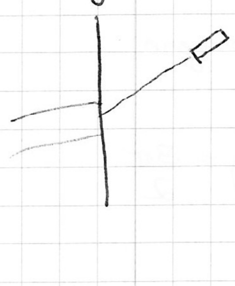
Rulings on ~~plate~~ ~~fresh~~ plastic

Can make holographically polymer material
Surface is sinusoidal looking





Symmetric equal amounts go into ± order



make groves asymmetric: blazing the grating
θ determines the λ that is preferentially diffracted in a particular order than another order.
Can also be silvered

Blazing a grating