95.538 Homework #7 28 April 2009 Meg Noah

15/20

4.1 The plates of a Fabry-Perot interferometer are ~~closed~~ coated with silver of such a thickness that for each plate, the reflectance is R=.9, the transmittance is 0.05=T, and the absorption A=0.05. Find the maximum and minimum transmittance of the interferometer. What is the value of the reflecting finesse and of the coefficient of finesse?

$$\mathcal{T}_{max} = \frac{T^2}{(1-R)^2} = \frac{0.05^2}{(1-.9)^2} = .25 \quad \mathcal{T}_{min} = \frac{T^2}{(1+R)^2} = \frac{.05^2}{(1+.9)^2} = 6.925\cdot 10^{-4}$$

$$\mathcal{F} = \frac{\pi\sqrt{R}}{(1-R)} = \frac{\pi\sqrt{0.05}}{(1-0.05)} = 0.74 \quad F = \frac{4R}{(1-R)^2} = \frac{4(0.05)}{(1-0.05)^2} = 0.22$$

29.78 2/4

4.2 Find the resolving power of the interferometer in Problem 4.1. if the plate separation is 1cm and the wavelength is 500 nm.

$$2N\pi = \frac{4\pi}{\lambda_0} nd\cos\theta + \delta_r \Rightarrow N = \frac{2d}{\lambda_0} \Rightarrow RP = N\mathcal{F} = \frac{2d\pi\sqrt{R}}{\lambda_0(1-R)} = 29600$$

4/6 ≈10

4.7 Calculate the reflectance of a quarter-wave antireflecting film of magnesium fluoride (n=1.35) coated on an optical glass surface of index 1.52. $n_1 = 1.35$ $n_T = 1.52$

$$R = \frac{(n_T - n_1^2)^2}{(n_T + n_1^2)^2} = \frac{(1.52 - 1.35^2)}{(1.52+1.35^2)} = 0.0082$$

3/3

4.8 Find the peak reflectance of a high-reflecting multilayer film consisting of index $n_L = 1.4$ and $n_H = 2.8$, eight layers, 4 of each, N=4

$$R = \left[\frac{(n_H/n_L)^{2N} - 1}{(n_H/n_L)^{2N}+1}\right]^2 = \left[\frac{(2.8/1.4)^{2\cdot 4}-1}{(2.8/1.4)^{2\cdot 4}+1}\right]^2 = \left[\frac{2^4-1}{2^4+1}\right]^2 = 0.78$$

8 +0.98 2/3

4.9 Fill in the steps in the derivation of the expression of the transfer matrix of a single film.

Start with the Boundary Conditions:

① $E_0 + E_0' = E_1 + E_1'$ ② $n_0(E_0 - E_0') = n_1(E_1 - E_1')$ ③ $E_1 e^{ikl} + E_1' e^{-ikl} = E_T$

④ $E_1 n_1 e^{ikl} - n_1 E_1' e^{-ikl} = n_T E_T$

multiply ③ by $n_1 \Rightarrow n_1 E_1 e^{ikl} + n_1 E_1' e^{-ikl} = n_1 E_T$
and add and subtract this to ④ to get:

⑤ $E_1 = \left(\frac{n_1 + n_T}{2n_1}\right) E_T e^{-ikl}$ ⑥ $E_1' = \left(\frac{n_1 - n_T}{2n_1}\right) E_T e^{+ikl}$

Substitute into ①

$$E_0 + E_0' = E_1 + E_1' = E_T e^{-ikl}\left(\frac{n_1+n_T}{2n_1}\right) + E_T e^{+ikl}\left(\frac{n_1 - n_T}{2n_1}\right) = E_T\left[\left(\frac{e^{-ikl}+e^{+ikl}}{2}\right) - \frac{n_T}{n_1}\left(\frac{e^{ikl}-e^{-ikl}}{2}\right)\right]$$

$$1 + \frac{E_0'}{E_0} = \left(\cos kl - i\frac{n_T}{n_1}\sin kl\right)\frac{E_T}{E_0} \quad ⑦$$

⇒

now substitute 5 and 6 into 2:

$$n_0 E_0 - n_0 E_0' = n_1 E_1 - n_1 E_1' = \left(\frac{n_1 + n_T}{2}\right) E_T e^{-ikl} - \left(\frac{n_1 - n_T}{2}\right) E_T e^{+ikl}$$

$$n_0 E_0 - n_0 E_0' = E_T \left\{ n_1 \frac{(e^{-ikl} - e^{+ikl})}{2} + n_T \left(\frac{e^{-ikl} + e^{+ikl}}{2}\right) \right\}$$

$$n_0 - n_0 \frac{E_0'}{E_0} = \frac{E_T}{E_0}\left\{ -n_1 \sin kl + n_T \cos kl \right\} \quad (8)$$

7 and 8 are equations (4.22) in book. Can be equivalently written in matrix form:

$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix} \frac{E_0'}{E_0} = \begin{bmatrix} \cos kl & (-i/n_1) \sin kl \\ -i n_1 \sin kl & \cos kl \end{bmatrix} \begin{bmatrix} 1 \\ n_T \end{bmatrix} \frac{E_T}{E_0}$$

let $r = \frac{E_0'}{E_0}$, $t = \frac{E_T}{E_0}$ and $M = \begin{bmatrix} \cos kl & -i/n_1 \sin kl \\ -i n_1 \sin kl & \cos kl \end{bmatrix}$

with n_1 = index of refraction $k = 2\pi/\lambda = 2\pi n_1/\lambda_0$ then

$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix} r = M \begin{bmatrix} 1 \\ n_T \end{bmatrix} t$$

$\frac{6}{6}$

5.1 Pinhole $\lambda = 600nm$ $d = 10m$ $\delta = 0.0$

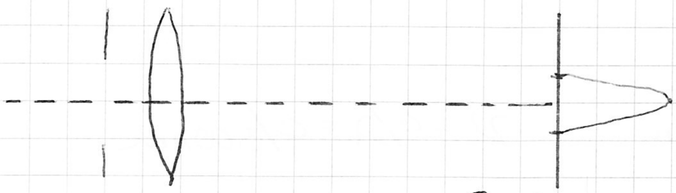
by Fowles Fraunhofer if $\frac{1}{2}\left(\frac{1}{d'} + \frac{1}{d}\right)\delta^2 \ll \lambda$

by Hecth Fraunhofer if $R > a^2/\lambda$ where $R = \min(d, d')$ $a = \max(\delta_i)$

$\lambda > a^2/R$

Distance to screen d'	1cm	2m	λ
Fowles $\frac{1}{2}(\frac{1}{d'} + \frac{1}{d})\delta^2 \ll \lambda$	$5.05 \cdot 10^{-6}m$ Fresnel	$3 \cdot 10^{-7}m$?	$6 \cdot 10^{-7}$
Hecht $\delta^2/d' > \lambda$	$1 \cdot 10^{-5}m$ Fresnel	$5 \cdot 10^{-7}m$ Fraunhofer	$6 \cdot 10^{-7}$

5.2 Collimated He-Ne $\lambda = 633nm$ $\delta_{slit} = 0.5mm$ $f = 50cm = r_0$



$\frac{kb}{2}\sin\theta = \beta$ $y = f\tan\theta$

$\theta = \sin^{-1}\left(\frac{2\beta}{kb}\right) = \sin^{-1}\left(\frac{\cancel{2}\lambda\beta}{\cancel{2}\pi b}\right)$

$\theta = \sin^{-1}\left(\frac{633 \cdot 10^{-9}}{.5 \cdot 10^{-3}}\frac{\beta}{\pi}\right) = \sin^{-1}\left(\frac{.001266\beta}{\pi}\right)$

Central maximum θ 0 1, $\frac{y}{\theta}$

	β (Hecht)	$\theta = \sin^{-1}(\frac{.001266\beta}{\pi})$	$y = f\tan\theta$	I
Central Max	0	0	0	I_0
1st Min	~~1.4303π~~ π	.001266	0.0633 cm	0
1st Max	1.4303π	~~rad~~ .0018 rad	0.0905 cm	$.0496 I_0$
2nd Min	2π	.002532	0.1266 cm	0
2nd Max	2.4590π	.003113	0.1556 cm	$.0168 I_0$

$\Delta_1 = |y_{Max0} - y_{min1}| = 0.0633cm$

$\Delta_2 = |y_{max0} - y_{max2}| = 0.0905cm$

$\Delta_3 = |y_{max0} - y_{min2}| = 0.1266cm.$

5.3 White light $\lambda = 650nm$ Coincide $\theta_1 = \theta_2$

4th max 3rd max

$b\sin\theta \approx (m + \frac{1}{2})\lambda$ max

$(3 + \frac{1}{2})\lambda_R = (4 + \frac{1}{2})\lambda$

$\lambda = \frac{7/2}{9/2}\lambda_R = 505nm$ (507nm)

5.5 Prove...
Hecht

$\nu = c/\lambda \qquad d\nu = (c/\lambda^2)\, d\lambda$

$d\nu = \frac{c\,\nu}{c\,\lambda} d\lambda$

$\left(\frac{\sin\beta}{\beta}\right)' = \frac{\beta\cos\beta - \sin\beta}{\beta^2} = 0 \qquad \frac{d\nu}{\nu} = \frac{d\lambda}{\lambda}$

$\beta\cos\beta - \sin\beta = 0$

$\beta = \tan\beta$

$\frac{d}{d\beta}\left(\frac{\sin\beta}{\beta}\right)^2 =$

5.13 grating has 1000 lines/mm $\qquad \nu = c/\lambda = c/500\text{nm}$

$\Delta\nu = 450\text{MHz}$ $\qquad \cancel{\frac{\Delta\lambda}{\lambda}}$

1000 lines/mm

$RP = \frac{\lambda}{\Delta\lambda} = \frac{\nu}{\Delta\nu} = \left(\frac{N}{L}\right) n L \qquad n\lambda = b\sin\theta$

$n = 1$

$L = RP$

$L = \frac{RP}{n\,(N/L)} = \frac{\nu}{\Delta\nu (N/L)\, n} = \frac{c}{\lambda\Delta\nu(N/L)\, n}$

$L = \frac{3\cdot 10^8}{\cancel{500}\,633 \cdot 10^{-9}\ 450\cdot 10^6\ (1000/10^{-3})\, n} = \left(\frac{\cancel{1.33}\ 1.05}{n}\right) m$

5.14

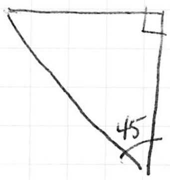
$\frac{\lambda}{\Delta\lambda} = RP = \left(\frac{N}{L}\right) L\, n$

$\Delta\lambda = \frac{\lambda}{(N/L)\, L\, n} = \frac{500\cdot 10^{-9}}{(1200/10^{-3})(.05)} = 8.3\cdot 10^{-12}\, m$

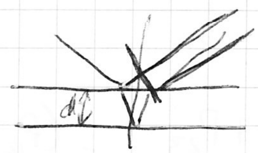
$.08\,Å$

May 18th Monday 8am

~~15) 1.2 In addition to Coulomb interaction, there exists another, called the hyperfine interaction, between the electron and the proton in the hydrogen atom. The Hamiltonian~~



Thin film



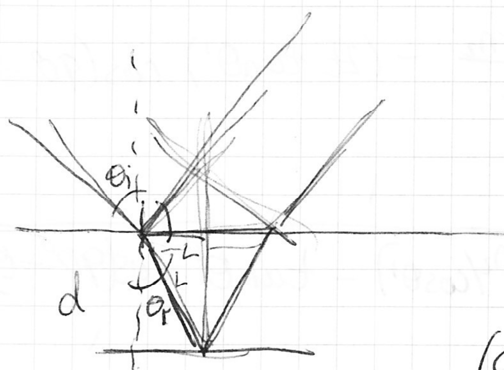
$\cancel{\Delta\phi = 2\pi}$

$\Delta L =$

$\frac{2d}{\cos\alpha}$

$\cancel{2\pi} m\lambda =$

$\cancel{2\pi} m(\lambda + 1/2) =$

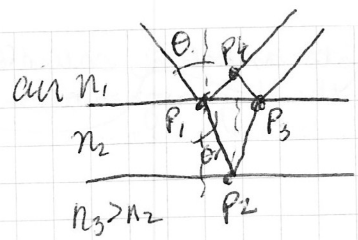


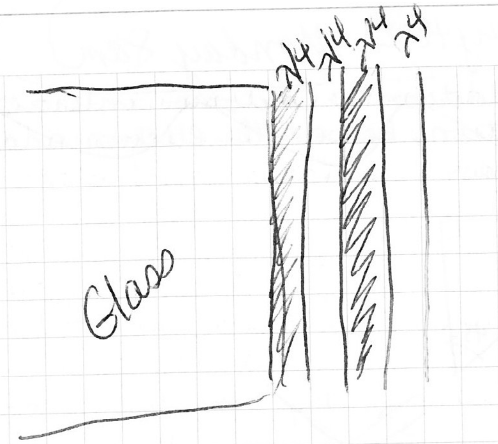
$(90 - \theta_1)$

$\frac{d}{x_1} =$

$\delta = \frac{2\pi}{\lambda_0} n(x_1 - x_2)$

$x_1 =$





$$\frac{\Phi_1}{2\pi} = \frac{\overline{P_1P_4}}{\lambda}$$

$$\frac{\Phi_2}{2\pi} = \frac{P_{12}+P_{23}}{\lambda_{n2}} = \frac{(P_{12}+P_{23})n_2}{\lambda}$$

$$\phi = \phi_2 - \phi_1$$

$$\phi = \left(\frac{2t}{\cos\theta'}\right)\frac{2\pi n_2}{\lambda} - (2t\tan\theta')\cos(90^\circ-\theta)\frac{2\pi}{\lambda}$$

$$t = \frac{\lambda\phi}{4\pi}\,\frac{1}{(n_2/\cos\theta') - \tan\theta'\cos(90^\circ-\theta)}$$

$\phi = 2\pi m$ constructive

$= 2\pi(m+\tfrac{1}{2})$ destructive

$3\cdot10^8\,ms^{-1}$

$\lambda = \frac{c}{\nu}$

$\nu = \frac{c}{\lambda} = \frac{3\cdot10^8\,ms^{-1}}{1\cdot10^{-9}m} =$

$\lambda = \frac{3\cdot10^8\,ms^{-1}}{13.56\cdot10^6\,s^{-1}} =$

nm to cm

$\bar{\nu} = 1/\lambda \quad nm \frac{10^{-9}m}{nm}\frac{cm}{10^{-2}m} = 10^{-7}$

$k = \frac{2\pi}{\lambda}$

$\bar{\nu} = 1/\lambda$

$\lambda = 1/\bar{\nu}$

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