Optics 95.538 Homework Set #6 $\frac{18}{20}$ Meg A. Noah

3.1 Calculate the interference pattern that would be obtained if ~~there were~~ three identical slits instead of two were used in Young's experiment

$$\left.\begin{aligned}\vec{E}_{(1)} &= \vec{E}_1 e^{i(\vec{k}_1\cdot\vec{r}-\omega t+\phi_1)}\\ \vec{E}_{(2)} &= \vec{E}_2 e^{i(\vec{k}_2\cdot\vec{r}-\omega t+\phi_2)}\\ \vec{E}_{(3)} &= \vec{E}_3 e^{i(\vec{k}_3\cdot\vec{r}-\omega t+\phi_3)}\end{aligned}\right\} \quad \vec{E} = \vec{E}_{(1)} + \vec{E}_{(2)} + \vec{E}_{(3)} \text{ and } |\vec{E}_{(1)}| = |\vec{E}_{(2)}| = |\vec{E}_{(3)}| = \sqrt{I_0}$$

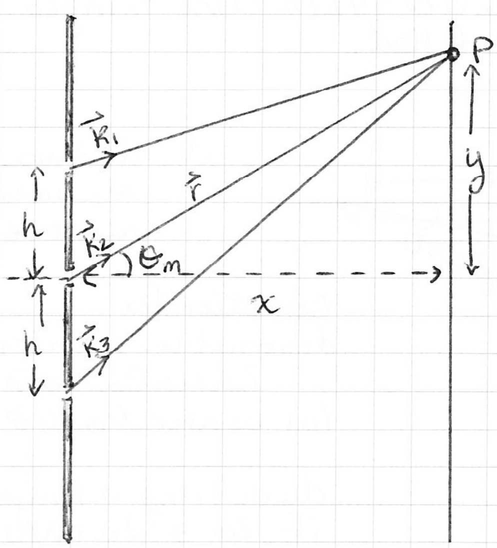
$$I = |\vec{E}|^2 = \vec{E}\cdot\vec{E}^* = (\vec{E}_{(1)} + \vec{E}_{(2)} + \vec{E}_{(3)})\cdot(\vec{E}^*_{(1)} + \vec{E}^*_{(2)} + \vec{E}^*_{(3)})$$

$$I = \vec{E}_{(1)}\cdot\vec{E}^*_{(1)} + \vec{E}_{(1)}\cdot\vec{E}^*_{(2)} + \vec{E}_{(1)}\cdot\vec{E}^*_{(3)} + \vec{E}_{(2)}\cdot\vec{E}^*_{(1)} + \vec{E}_{(2)}\cdot\vec{E}^*_{(2)} + \vec{E}_{(2)}\cdot\vec{E}^*_{(3)} + \vec{E}_{(3)}\cdot\vec{E}^*_{(1)} + \vec{E}_{(3)}\cdot\vec{E}^*_{(2)} + \vec{E}_{(3)}\cdot\vec{E}^*_{(3)}$$

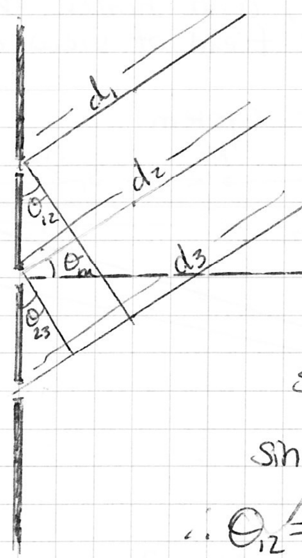
$$I = |E_1|^2 + |E_2|^2 + |E_3|^2 + 2\vec{E}_1\cdot\vec{E}_2^*\cos\theta_{12}(\vec{r}_{12}) + 2\vec{E}_2\cdot\vec{E}_3^*\cos\theta_{23}(\vec{r}_{23}) + 2\vec{E}_1\cdot\vec{E}_3^*\cos\theta_{13}(\vec{r}_{13})$$

$$I = I_0 + I_0 + I_0 + 2I_0\cos\theta_{12} + 2I_0\cos\theta_{23} + 2I_0\cos\theta_{13}$$

$$I = 3I_0 + 2I_0\cos\theta_{12} + 2I_0\cos\theta_{23} + 2I_0\cos\theta_{13}$$



$x \gg y$



$\theta_{12} = \theta_{23} = \theta = k(\Delta d)$

$\theta_{12} = k(d_2 - d_1) = kh\sin\theta_m$

$\theta_{23} = k(d_3 - d_2) = kh\sin\theta_m$

$\theta_{13} = k(d_3 - d_1) = k2h\sin\theta_m$

small angle $\theta_m \sim \sin\theta_m \sim \tan\theta_m$

$\sin\theta_m \sim \tan\theta_m \sim \frac{y}{x}$

$\therefore \theta_{12} = \theta_{23} = \frac{khy}{x} = \theta$

$\theta_{31} = \frac{2khy}{x} = 2\theta$

$$I = 3I_0 + 2I_0\cos\theta_{12} + 2I_0\cos\theta_{23} + 2I_0\cos\theta_{31}$$

$$\frac{I}{I_0} = 3 + 2\cos\left(\frac{khy}{x}\right) + 2\cos\left(\frac{khy}{x}\right) + 2\cos\left(\frac{2khy}{x}\right)$$

$$\frac{I}{I_0} = 3 + 4\cos\left(\frac{khy}{x}\right) + 2\cos\left(\frac{2khy}{x}\right)$$

$$\frac{I}{I_0} = 3 + 4\cos\theta + 2\cos 2\theta, \quad \theta = \frac{khy}{x}$$

NOTE BOOK

3.2 In the two-slit interference experiment of the Young type, the aperture-to-screen distance is 2m and the wavelength is 600nm. If it is desired to have a fringe spacing of 1mm, what is the required slit separation?

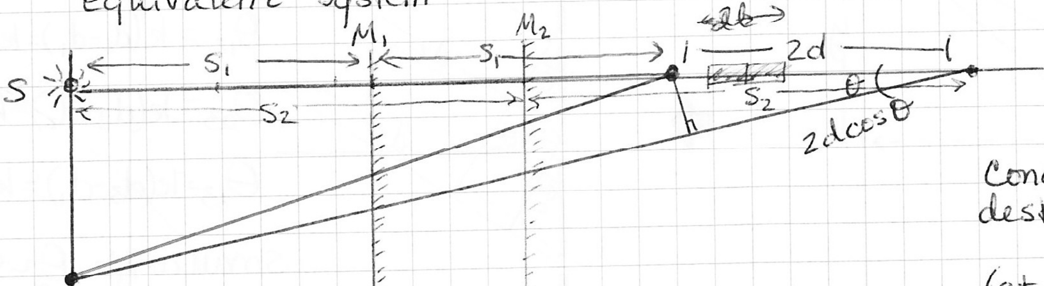
$x = 2m \quad \lambda = 600nm \quad \Delta y = 1mm \qquad y = 0, \pm\frac{\lambda x}{h}, \pm\frac{2\lambda x}{h}$, etc.

$$\Delta y = \frac{\lambda x}{h} \Rightarrow h = \frac{\lambda x}{\Delta y} = \frac{(600\cdot 10^{-9}m)(2m)}{(1\cdot 10^{-3}m)} = 1200\cdot 10^{-6}m = 1.2\cdot 10^{-3}m = 1.2mm$$

3.6 A Michelson interferometer can be used to determine the index of refraction of a gas. The gas is made to flow into an evacuated glass cell of length l placed in one arm of the interferometer. The interference fringes are counted as they move across the view aperture when the gas flows into the cell.

Show that the effective optical path difference of the light beam for the full cell vs. the evacuated cell is $2l(n-1)$ where n is the index of refraction, and hence that a number $N = 4l(n-1)/\lambda$ fringes move across the field of view as the cell is filled.

equivalent system



d = distance M_2 moves relative to M_1

Condition for destructive interference:

$2d\cos\theta_m = m\lambda$

(at center $\theta_m = 0 \Rightarrow 2d = m\lambda$)

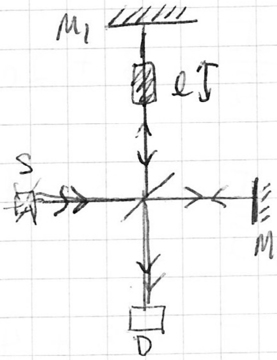
dark fringe ⇒ bright fringe if $2d \rightarrow 2d + \lambda/2$

$\therefore \Delta L = \frac{\lambda}{2}$ = optical path length difference

each fringe $= \Delta L = \frac{\lambda}{2}$

N fringes: $\frac{N\lambda}{2} = 2l(n-1)$

$$N = \frac{4l(n-1)}{\lambda}$$



How many fringes would be counted if the gas were air ($n = 1.0003$) for a 10 cm cell using yellow sodium light $\lambda = 590nm$?

$$N = \frac{4(10\cdot 10^{-2}m)(1.0003-1)}{(590\cdot 10^{-9}m)} = 2{,}033{,}898 \text{ fringes}$$

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With gas cell

path difference $= 2ln + d_1 - d_2$

if $d_1 - d_2$ is constant

Difference Optical Path w/ and w/o gas

$2ln_1 - 2ln_2 = 2l(n_1 - n_2) = 2l(n-1)$

n_1 air w/ gas

3.8 What is the line width in hertz an in nanometers of the light from a He-Ne laser whose coherence length is 5 km? The wavelength is 633 nm.

$\lambda = 633 \cdot 10^{-9} m \qquad l_c = 5 \cdot 10^3 m$

(3.35) $\Delta\nu = \frac{c}{l_c} = \frac{3 \cdot 10^8 m/s}{5 \cdot 10^3 m} = 60{,}000\ Hz$ (3.36) $\Delta\lambda = \frac{\lambda^2}{l_c} = \frac{(633 \cdot 10^{-9} m)^2}{(5 \cdot 10^3 m)} = 8 \cdot 10^{-17} m$

3.9 What is the transverse coherence width of sunlight? The apparent angular diameter of the sun is 0.5° and the mean effective wavelength is 600 nm.

(3.44) $l_t = \frac{1.22\lambda}{\theta_s} = \frac{(1.22)(600 \cdot 10^{-9} m)}{(.5°)(\pi\ rad/180°)} = 8.4 \cdot 10^{-5} m$

3.12 Calculate the power spectrum of a damped wave train:

$f(t) = A \exp(-at - i\omega_0 t) \qquad t \geq 0$

$f(t) = 0 \qquad t < 0$

$$g(\omega) = \frac{1}{\sqrt{2\pi}} \int_0^\infty f(t) e^{i\omega t} dt = \frac{A}{\sqrt{2\pi}} \int_0^\infty e^{-at - i\omega_0 t - i\omega t} dt = \frac{A}{\sqrt{2\pi}} \int_0^\infty e^{-bt} dt$$

where $b = a + i\omega_0 + i\omega$

$$g(\omega) = \frac{A}{\sqrt{2\pi}} \frac{e^{-bt}}{(-b)} \Big|_0^\infty = \frac{A}{\sqrt{2\pi}} \frac{(e^{-\infty} - e^0)}{(-b)} = \frac{A}{\sqrt{2\pi}\, b} = \frac{A}{\sqrt{2\pi}(a + i\omega_0 + i\omega)}$$

$$G(\omega) = |g(\omega)|^2 = \frac{A}{\sqrt{2\pi}(a + i\omega_0 + i\omega)} \frac{A}{(a - i\omega_0 - i\omega)\sqrt{2\pi}}$$

$$G(\omega) = \frac{A^2}{2\pi} \frac{1}{a^2 - (\omega_0 - \omega)^2}$$

or is it

$$G(\omega) = \frac{A^2}{2\pi} \left[\frac{1}{a + i(\omega_0 + \omega)}\right]^2 = \frac{A^2}{2\pi} \frac{1}{a^2 + 2ia(\omega_0 + \omega) - (\omega_0 + \omega)^2}$$