20/2

Homework Set #5 95.538 24 March 2009 Meg Noah

2.14 Find the critical angle for internal reflection in water ($n=1.33$) and diamond ($n=2.42$). (relative to air)

$\theta_c = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{1.33}\right) = 48.75°$ water to air

$\theta_c = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{2.42}\right) = 24.41°$ diamond to air

2.15 Find the Brewster angle for external reflection in water and diamond.

$\theta_B = \tan^{-1}(n) = \tan^{-1}(2.42) = 67.55°$ diamond

$\theta_B = \tan^{-1}(n) = \tan^{-1}(1.33) = 53.06°$ water

2.16 Find the reflectance for both TE and TM polarizations at an angle of incidence of 45° for water and diamond. (external?)

$$r_s = \frac{\cos\theta - \sqrt{n^2-\sin^2\theta}}{\cos\theta + \sqrt{n^2-\sin^2\theta}} = \frac{\cos 45° - \sqrt{1.33^2-\sin^2 45°}}{\cos 45° + \sqrt{1.33^2-\sin^2 45°}} = \frac{.707 - 1.12645}{.707 + 1.12645} = -0.230453$$

$R_s = |r_s|^2 = 0.053$ for water TE Reflectance

$$r_p = \frac{-n^2\cos\theta + \sqrt{n^2-\sin^2\theta}}{n^2\cos\theta + \sqrt{n^2-\sin^2\theta}} = \frac{-(1.33^2)\frac{1}{\sqrt{2}} + \sqrt{1.33^2-\sin^2 45°}}{(1.33^2)\frac{1}{\sqrt{2}} + \sqrt{1.33^2-\sin^2 45°}} = \frac{-1.251+1.12645}{1.251+1.12645} = -.058$$

$R_p = |r_p|^2 = 0.00274$ for water TM Reflectance

$$r_s = \frac{\cos\theta - \sqrt{n^2-\sin^2\theta}}{\cos\theta + \sqrt{n^2-\sin^2\theta}} = \frac{.707 - \sqrt{2.42^2 - .5}}{.707 + \sqrt{2.42^2 - .5}} = \frac{.707 - 2.3144}{.707 + 2.3144} = -0.532$$

$R_s = |r_s|^2 = 0.283$ for diamond TE Reflectance

$$r_p = \frac{-n^2\cos\theta + \sqrt{n^2-\sin^2\theta}}{n^2\cos\theta + \sqrt{n^2-\sin^2\theta}} = \frac{-(2.42^2)(.707) + 2.3144}{(2.42^2)(.707) + 2.3144} = -0.2829$$

$R_p = |r_p|^2 = .08$ for diamond TM Reflectance

2.17 The critical angle for internal reflection in a certain substance is exactly 45°. What is the Brewster angle for external reflection?

$\sin 45° = \frac{1}{\sqrt{2}} = \frac{1}{n}$

$\theta_c = 45° = \sin^{-1}\left(\frac{1}{n}\right) \Rightarrow n = \sqrt{2} = 1.414$

$\theta_B = \tan^{-1}(n) = \tan^{-1}\sqrt{2} = 54.74°$

2.19 A beam of light is totally reflected in a 45-90-45 glass prism ($n=1.5$) [Fig 2.15] The wavelength of the light $\lambda = 500$ nm. At what distance from the surface is the amplitude of the evanescent wave 1/e of its value at the surface?

$\alpha = k''\sqrt{\frac{\sin^2\theta}{n^2} - 1}$

$\frac{E_{tran}}{E} = \frac{1}{e} = e^{-\alpha|y|} \Rightarrow |y| = +\frac{1}{\alpha} =$ 337.6 nm $=$ .0003376 mm

$\alpha = \frac{nk}{\sin\theta}\sqrt{\frac{\sin^2\theta}{n^2} - 1} = \frac{n2\pi}{\lambda\sin\theta}\sqrt{\frac{\sin^2\theta}{n^2} - 1} = \frac{(1.5)2\pi}{(1.5)(500\,nm)}\sqrt{\frac{\sin^2 45°\,1.5^2}{}-1} = .00888$

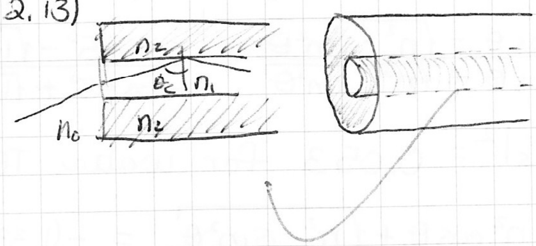
I can only get the answer in book with $\boxed{k'' = \frac{n2\pi}{\lambda}}$

By what factor is the intensity of the evenescent wave reduced at a distance of 1mm from the surface?

$$\frac{E_{tran}}{E} = e^{-\alpha y} \Rightarrow e^{-2960}$$

$$\alpha = .00296\,nm^{-1} \qquad \alpha y = (.00296\,nm^{-1})\left(\frac{1\cdot 10^{9}\,nm}{m}\right)\left(\frac{1m}{1\cdot 10^{3}\,mm}\right)(1mm) = 2960$$

2.20 Show that the acceptance angle for a glass-fiber waveguide is given by $\alpha = \sin^{-1}\sqrt{n_1^2 - n_2^2}$ where n_1 and n_2 are the indices of refraction of the fiber and the cladding material, respectively, and the external medium is air, $n_0 = 1$ (see figure 2.13)



$$\sin\alpha' = \cos\theta_c = \sqrt{1-\sin^2\theta_c}$$
$$\sin\alpha' = \sqrt{1-(n_2/n_1)^2}$$
$$n_1 \sin\alpha' = n_0 \sin\alpha$$
$$\frac{n_0}{n_1}\sin\alpha = \sqrt{1-(n_2/n_1)^2}$$

$$n_0 \sin\alpha = \sqrt{n_1^2 - n_2^2} \Rightarrow n_0 = 1 \Rightarrow \sin\alpha = \sqrt{n_1^2-n_2^2} \Rightarrow \alpha = \sin^{-1}\sqrt{n_1^2-n_2^2}$$

2.21 Fill in the steps leading to the equation: $\tan\Delta/2 = \cos\theta\sqrt{\sin^2\theta - n^2}\,/\sin^2\theta$ for the phase difference in total internal reflection discussed in Sec. 2.10.

$$ae^{i\alpha} = \cos\theta + i\sqrt{\sin^2\theta - n^2} = a\cos\alpha + ia\sin\alpha \qquad be^{i\beta} = n^2\cos\theta + i\sqrt{\sin^2\theta - n^2} = b\cos\beta + ib\sin\beta$$

$$\delta_s = 2\alpha \quad \delta_p = 2\beta \qquad \frac{\Delta}{2} = \frac{\delta_p - \delta_s}{2} = \beta - \alpha \Rightarrow \tan(\Delta/2) = \tan(\beta-\alpha) = -\sin\alpha\cos\alpha + \sin\beta\cos\beta$$

$$\cancel{\tan(\Delta/2) = \sin\alpha\cos\alpha + \sin\beta\cos\beta = \left(\frac{\sqrt{\sin^2-n^2}}{a}\right)\left(\frac{\cos\theta}{a}\right) + \left(\frac{\sqrt{\sin^2-n^2}}{b}\right)\left(\frac{n^2\cos\theta}{b}\right)}$$

$$\cancel{\tan(\Delta/2) = \cos\theta\sqrt{\sin^2\theta-n^2}\left(\frac{n^2}{b^2} - \frac{1}{a^2}\right)} \qquad \cancel{a^2 = (\cos\theta + i\sqrt{\sin^2\theta-n^2})(\cos\theta - i\sqrt{\sin^2\theta-n^2}) = \cos^2\theta + \sin^2\theta - n^2 = 1-n^2}$$

$$\tan(\Delta/2) = \tan\left(\frac{\delta_p}{2} - \frac{\delta_s}{2}\right) = \frac{\tan\delta_p/2 - \tan\delta_s/2}{1 - \tan\delta_s/2\,\tan\delta_p/2} = \frac{\tan\beta - \tan\alpha}{1 + \tan\beta\tan\alpha}$$

$$\tan\beta = \frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta} \qquad \tan\alpha = \frac{\sqrt{\sin^2\theta-n^2}}{\cos\theta}$$

$$\tan(\Delta/2) = \frac{\tan\beta - \tan\alpha}{1+\tan\beta\tan\alpha} = \frac{\sqrt{\sin^2\theta-n^2}\left(\frac{1}{n^2\cos\theta} - \frac{1}{\cos\theta}\right)}{1 + \frac{(\sin^2\theta-n^2)}{n^2\cos^2\theta}} = \frac{\cos\theta\sqrt{\sin^2\theta - n^2}\left(\frac{1}{n^2}-1\right)}{\cos^2\theta + \frac{\sin^2\theta}{n^2} - 1}$$

$$\tan(\Delta/2) = \frac{\cos\theta\sqrt{\sin^2\theta-n^2}\left(\frac{1}{n^2}-1\right)}{\sin^2\theta\left(\frac{1}{n^2}-1\right)} = \frac{\cos\theta\sqrt{\sin^2\theta - n^2}}{\sin^2\theta}$$

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Algebra of the Fresnel Coefficients

TE Polarizations

$$E + E' = E'' \quad (1)$$
$$-H\cos\theta + H'\cos\theta = -H''\cos\phi \quad (2)$$
$$-kE\cos\theta + k'E'\cos\theta = -k''E''\cos\phi \quad (3)$$

$k' = k$, $k'' = nk$ substitute:

$$E + E' = E'' \quad (1a)$$
$$-H\cos\theta + H'\cos\theta = -H''\cos\phi \quad (2a)$$
$$-kE\cos\theta + kE'\cos\theta = -nkE''\cos\phi \quad (3a)$$

$$r_s = \left[\frac{E'}{E}\right]_{TE}$$

substitute (1a) into (3a)

$$(-E + E')\cancel{k}\cos\theta = -n\cancel{k}(E' + E)\cos\phi$$

rearrange:

$$E'(\cos\theta + n\cos\phi) = E(\cos\theta - n\cos\phi)$$

$$r_s = \left[\frac{E'}{E}\right]_{TE} = \frac{\cos\theta - n\cos\phi}{\cos\theta + n\cos\phi} \quad (2.54)$$

Use: $n = \sin\theta/\sin\phi$

$$r_s = \frac{\cos\theta - \sin\theta\cos\phi/\sin\phi}{\cos\theta + \cos\phi\sin\theta/\sin\phi}$$

$$r_s = \frac{\sin\phi\cos\theta - \cos\phi\sin\theta}{\sin\phi\cos\theta + \cos\phi\sin\theta}$$

$$r_s = \frac{-\sin(\theta-\phi)}{\sin(\theta+\phi)} \quad (2.56)$$

$$n\cos\phi = \sqrt{n^2\cos^2\phi} = \sqrt{n^2(1-\sin^2\phi)}$$
$$n\cos\phi = \sqrt{n^2 - \sin^2\theta}$$

$$r_s = \frac{\cos\theta - n\cos\phi}{\cos\theta + n\cos\phi} = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} \quad (2.58)$$

TM Polarization

$$H - H' = H'' \quad (4)$$
$$kE - k'E' = k''E'' \quad (5)$$
$$E\cos\theta + E'\cos\theta = E''\cos\phi \quad (6)$$

$$H - H' = H'' \quad (4a)$$
$$kE - kE' = nkE'' \quad (5a) \Rightarrow E - E' = nE''$$
$$E\cos\theta + E'\cos\theta = E''\cos\phi \quad (6a)$$

$$r_p = \left[\frac{E'}{E}\right]_{TM}$$

Substitute (5a) into (6a)

$$(E + E')\cos\theta = (E - E')(\cos\phi/n)$$

rearrange:

$$E'(\cos\theta + \cos\phi/n) = E(\cos\phi/n - \cos\theta)$$

$$r_p = \left[\frac{E'}{E}\right]_{TM} = \frac{\cos\phi/n - \cos\theta}{\cos\theta + \cos\phi/n}$$

$$r_p = \frac{-n\cos\theta + \cos\phi}{n\cos\theta + \cos\phi} \quad (2.55)$$

use Snell's Law

$$r_p = \frac{-\sin\theta\cos\theta/\sin\phi + \cos\phi}{\sin\theta\cos\theta/\sin\phi + \cos\phi}$$

$$r_p = \frac{-\sin\theta\cos\theta + \sin\phi\cos\phi}{\sin\theta\cos\theta + \sin\phi\cos\phi}$$

$$r_p = -\frac{\tan(\theta-\phi)}{\tan(\theta+\phi)} \quad (2.57)$$

use $n\cos\phi = \sqrt{n^2 - \sin^2\theta}$ in 2.55

$$r_p = \frac{-n^2\cos\theta + \cos\phi\, n}{n^2\cos\theta + n\cos\phi}$$

$$r_p = \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} \quad (2.59)$$

$t_s, t_p \Rightarrow$

from side 1

TE Polarization

$E + E' = E''$ (1a)

$(E + E')\cos\theta = -nE''\cos\phi$ (3a)

~~substs~~

$$t_s = \left[\frac{E''}{E}\right]_{TE}$$

Substitute (1a) into (3a)

$$(-E + E'' - E)\cos\theta = -nE''\cos\phi$$

$$E''(\cos\theta + n\cos\phi) = E(2\cos\theta)$$

$$t_s = \left[\frac{E''}{E}\right]_{TE} = \frac{2\cos\theta}{\cos\theta + n\cos\phi}$$

$$t_s = \frac{2\cos\theta\sin\phi}{\sin\phi\cos\theta + n\sin\phi\cos\phi}$$

$$n = \sin\theta/\sin\phi \rightarrow n\sin\phi = \sin\theta$$

$$t_s = \frac{2\cos\theta\sin\phi}{\sin\phi\cos\theta + \cos\phi\sin\theta}$$

$$t_s = \frac{2\cos\theta\sin\phi}{\sin(\theta + \phi)} \quad (2.56)$$

TM Polarization

$E - E' = nE''$ (5a)

$(E + E')\cos\theta = E''\cos\phi$ (6a)

$$t_p = \left[\frac{E''}{E}\right]_{TM}$$

Substitute (5a) into (6a)

$$(E + E - nE'')\cos\theta = E''\cos\phi$$

$$E''(-n\cos\theta - \cos\phi) = -2E\cos\theta$$

$$t_p = \left[\frac{E''}{E}\right] = \frac{2\cos\theta}{n\cos\theta + \cos\phi}$$

$$t_p = \frac{2\cos\theta\sin\phi}{n\sin\phi\cos\theta + \sin\phi\cos\phi}$$

$$n\sin\phi = \sin\theta$$

$$t_p = \frac{2\cos\theta\sin\phi}{\sin\theta\cos\theta + \sin\phi\cos\phi}$$

$$t_p = \frac{2\cos\theta\sin\phi}{\sin(\phi + \phi)\cos(\theta - \phi)} \quad (2.57)$$