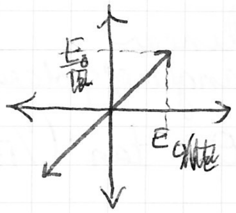
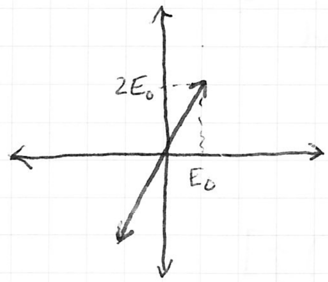
Optics Homework Set #4 Meg Noah

$\frac{19}{20}$

2.6 a) $\phi = 0 \quad b = 1 \qquad \vec{E} = E_0[\hat{i}\cos(kz-\omega t) + \hat{j}\cos(kz-\omega t)]$



b) $\phi = 0 \quad b = 2 \qquad \vec{E} = E_0[\hat{i}\cos(\cancel{\ } kz-\omega t) + \hat{j}\, 2\cos(kz-\omega t)]$

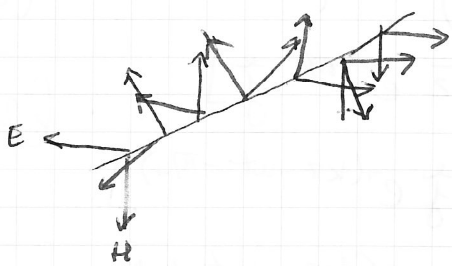



angle ?

c) $\phi = \pi/2 \quad b = -1$ circularly polarized

$\vec{E} = E_0[\hat{i}\cos(kz-\omega t) - \hat{j}(\cos(kz\ \omega t + \pi/2)$

$\vec{E} = E_0[\hat{i}\cos(kz-\omega t) - \hat{j}\sin(\cancel{\cos}\, kz-\omega t) = E_0 e^{i(kz-\omega t)}[\hat{i} - i\hat{j}]$




d) $\phi = \pi/4 \quad b = 1$

$\vec{E} = E_0[\hat{i}\cos(kz-\omega t) + \hat{j}\cos(\cancel{kz} - \omega t + \pi/4)$

$\vec{E} = E_0 e^{i(kz-\omega t)}[\hat{i} + e^{i\pi/4}\hat{j}]$

$e^{i\pi/4} = \cos\pi/4 + i\sin\pi/4 = \sqrt{2}/2 + i\sqrt{2}/2$

$\vec{E} = E_0 e^{i(kz-\omega t)}\left[\hat{i} + \left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)\hat{j}\right] = E_0 \frac{\sqrt{2}}{2} e^{i(kz-\omega t)}[\sqrt{2}\,\hat{i} + (1+i)\hat{j}]$

2.8 Describe type of polarization

$\begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$ $\quad \vec{E} = E_0(\hat{i} + \sqrt{3}\,\hat{j})e^{i(kz-\omega t)}$ $\quad$ linear

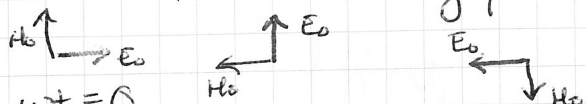
angle of polarization

$\theta = \tan^{-1}(1/\sqrt{3}) = 30°$

$\begin{bmatrix} i \\ -1 \end{bmatrix}$ $\quad \vec{E} = E_0(e^{i\pi/2}\,\hat{i} - \hat{j})\,e^{i(kz-\omega t)}$

$\vec{E} = E_0(e^{i(kz-\omega t+\pi/2)}\,\hat{i} - e^{i(kz-\omega t)}\,\hat{j})$

$\phi = \pi/2 \Rightarrow$ circularly polarized




$\omega t = 0$ $\quad \omega t = \pi/2$ $\quad \omega t = \pi$

$z = 0$

$\begin{bmatrix} 1-i \\ 1+i \end{bmatrix}$ $\quad \vec{E} = E_0\,e^{i(kz-\omega t)}[(1-i)\hat{i} + (1+i)\hat{j}]$

$$(1+i) = \sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \sqrt{2}(\cos\pi/4 + i\sin\pi/4) = \sqrt{2}\,e^{i\pi/4}$$

$$(1-i) = \sqrt{2}\left(\frac{\sqrt{2}}{2} - \frac{i\sqrt{2}}{2}\right) = \sqrt{2}\left(\cos(-\pi/4) + i\sin\left(\frac{-\pi}{4}\right)\right) = \sqrt{2}\,e^{i\pi/4}$$

$$\vec{E} = E_0 e^{i(kz-\omega t)}\left[\sqrt{2}\,e^{i\pi/4}\,\hat{i} + \sqrt{2}\,e^{-i\pi/4}\,\hat{j}\right]$$

$$\vec{E} = E_0\sqrt{2}\left[\hat{i}\,e^{i(kz-\omega t+\pi/4)} + \hat{j}\,e^{i(kz-\omega t-\pi/4)}\right]$$

left circularly polarized

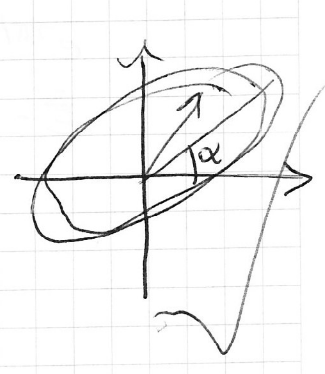
2.9 $\quad E_x = E_{0x}\cos(kz-\omega t) \quad E_y = E_{0y}\cos(kz-\omega t+\Delta)$

$$\frac{E_y}{E_{0y}} = \cos(kz-\omega t)\cos\Delta - \sin(kz-\omega t)\sin\Delta$$

$$\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}}\cos\Delta = -\sin(kz-\omega t)\sin\Delta$$

$$\sin(kz-\omega t) = [1-(E_x/E_{0x})^2]^{1/2}$$

$$\left(\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}}\cos\Delta\right)^2 = \left[1-\left(\frac{E_x}{E_{0x}}\right)^2\right]\sin^2\Delta$$




$$\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 - 2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\Delta = \sin^2\Delta$$

to write as ellipse condition is

$$\tan 2\alpha = \frac{2E_{0x}E_{0y}\cos\Delta}{E_{0x}^2 - E_{0y}^2} \Rightarrow \left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 = 1$$

2.10 $I_{out} = ?$ $I_{in}$ = unpol $= \begin{bmatrix} E_x \\ E_y \end{bmatrix}$ $Q = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$ $L = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$I_{out} = Q L I_{in} = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} E_x+E_y \\ E_x+E_y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ -i \end{bmatrix}(E_x+E_y)$$

RH Circularly Polarized

$$I_{out} = L Q I_n = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} E_x \\ -iE_y \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} E_x - iE_y \\ E_x - iE_y \end{bmatrix} = \frac{1}{2}(E_x - iE_y)\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$ linearly polarized light

2.11 verify $\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$ Circ polarizer

$I_{in}$ = Right-Circ-Polarized $= \begin{bmatrix} 1 \\ -i \end{bmatrix}$

$$I_{out} = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -i \end{bmatrix} = \begin{bmatrix} 1-(i)^2 \\ -i-i \end{bmatrix} = \begin{bmatrix} 2 \\ -2i \end{bmatrix} = 2\begin{bmatrix} 1 \\ -i \end{bmatrix}$$ Right Circ.

$I_{in}$ = Left Circ Polarized $= \begin{bmatrix} 1 \\ i \end{bmatrix}$

$$I_{out} = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}\begin{bmatrix} 1 \\ i \end{bmatrix} = \begin{bmatrix} 1+i^2 \\ -i+i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ "opaque" no transmission.

2.13 Eigenvalues + Eigenvectors

$LP = \frac{1}{2}\begin{bmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{bmatrix}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \lambda \begin{bmatrix} A \\ B \end{bmatrix}$

$$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \lambda\begin{bmatrix} A \\ B \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = 0$$

$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)^2 - 1 = 0$$

$$1 - 2\lambda + \lambda^2 - 1 = 0$$

$$\lambda(\lambda - 2) = 0$$

eigenvalues $\lambda = 0, 2$

eigenvectors:

$\lambda = 0$

$$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = 0$$

$$\tfrac{1}{2}(A+B) = 0 \Rightarrow A = -B$$

$\begin{bmatrix} A \\ B \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ when $\lambda = 0$ (normalized)

$\lambda = 2$

$$\frac{1}{2}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = 0$$

$$\tfrac{1}{2}(-A+B) = 0 \Rightarrow A = B$$

$\begin{bmatrix} A \\ B \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ when $\lambda = 2$ (normalized)