There's probably something wrong with this proof, but it was the best I could do as an undergraduate…

Proof that if $c^n = a^n + b^n$ then $a,\ b,$ and c can't all be integers. This proof shows that the contrary assumption leads to contradiction. Proof is to show that equations would only hold if $a,\ b,$ and c (integers) had a common integer factor k and therefore the equation can't be satisfied because k could be factored out so that $a,\ b,$ and c no longer satisfied the condition of having a common factor.

$$c < a + b$$
$$c^3 = a^3 + b^3$$
$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$
$$(a+b)^3 - c^3 = 3ab(a+b) \qquad \text{equation (A)}$$
$$(a+b)^3 - c^3 = 3z$$

z is also an integer.

When is the difference between two integer cubes an integer multiple of 3?

$$c < a + b$$
$$c^3 = a^3 + b^3$$
$$(c+1)^3 = c^3 + 1^3 + 3c(c+1) \text{ doesn't work}$$
$$(c+2)^3 = c^3 + 2^3 + 3c(c+2) \text{ doesn't work}$$
$$(c+3)^3 = c^3 + 3^3 + 3c(c+3) \text{ works!}$$
$$so$$
$$(a+b) = (c+3f)$$
$$(c+3f)^3 = c^3 + 3^3 f^3 + 3c3f(c+3f)$$
$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$
$$\cancel{a^3 + b^3} + 3ab(a+b) = \cancel{c^3} + 3^3 f^3 + 3c3f(c+3f)$$
$$3ab(a+b) = 3^3 f^3 + 3c3f(c+3f)$$
$$ab(a+b) = 3^2 f^3 + 3cf(c+3f)$$
$$\therefore \text{ either } a, b, \text{ or } a+b \text{ have to be a factor of 3}$$

If $\left(a+b\right)$ is a factor of 3, then c is a factor of $c = \left(a+b\right) - 3f$ is a factor of 3 and equation A can not be satisfied.

If a is a factor of 3, then the difference between

$a = 3g$

$c^3 = (3g)^3 + b^3$

$c = b + ?$

$(b+?)^3 = b^3 + (3g)^3$

$(b+1)^3 = b^3 + 1^3 + 3b(b+1)$ doesn't work

$(b+2)^3 = b^3 + 2^3 + 3b(b+2)$ doesn't work

$(b+3)^3 = b^3 + 3^3 + 3b(b+3)$ works!

$\therefore c = b + 3h$

$a + b = c + 3f$

$3g + b = b + 3h + 3f$

$\therefore b = 3i$

$\therefore c = b + 3h = 3i + 3h = 3j$

Since in order to satisfy the equation $a,\ b,$ and c all have to be factors of 3, the equation can’t be satisfied by integers.

Finally, use pascal’s triangle to solve for general n since

$c < a + b$

$c^n = a^n + b^n$

$(a+b)^n = a^n + b^n + nab\{...\}$

to show that n would have to be a common factor of $a,\ b,$ and c to satisfy the $c^n = a^n + b^n$ relation.