$1\,amu = 1.66054 \cdot 10^{-27}\,kg = .9314943\,GeV$

$20\,amu = .3321079 \cdot 10^{-25}\,kg = 18.629886\,GeV$

9.1 Laser transition ~~near~~ (Ne) $\lambda_0 = 6328\,Å$ is Doppler broadened by the thermal motion of the atoms at a gas $T \sim 300°C$ (assume pure Ne^{20}) $V = 1\,cm^2$ $T = 573\,K$

$\nu_0 = 4.743 \cdot 10^{14}\,Hz$

a) $\Delta\nu_D = \left[\frac{8kT \ln 2}{Mc^2}\right]^{1/2} \nu_0 = 1.81\,GHz$

b) $N_{BB} = \frac{8\pi\nu^2 \Delta\nu}{c^3} V = 3.793 \cdot 10^{8}$ modes

c) $FSR = c/2d = 150\,MHz$

$N_L = \frac{1.81}{0.15} = 12$ (or 13) $\times 2$ for both polarizations

d) $Probability = \frac{N_L}{N_{BB}} = 3.173 \cdot 10^{-8}$ ($\times 2$ ↓)

e) $E_2 = 166{,}658.484\,cm^{-1} = 20.66\,eV$

$E_1 = 150{,}855.7\,cm^{-1} = 18.70\,eV$

f) $\eta_{QE} = \frac{1.96}{20.66} = \frac{E_2 - E_1}{E_2} = 9.47\%$ (eq. 9.2.1)

g) $g(\nu_D) = \left(\frac{4\ln 2}{\pi}\right)^{1/2} \frac{1}{\Delta\nu_D} = 5.193 \cdot 10^{-10}\,s$

g) use $A_{21} = 6.56 \cdot 10^6\,sec^{-1}$

stimulated emission cross-section:

$\sigma = A_{21} \frac{\lambda^2}{8\pi} g(\nu_0) = 5.43 \cdot 10^{-13}\,cm^2$ (eq. 7.47)

h) If the density of excited Ne atoms in State 1 ($J_1 = 2$) is $10^{10}\,cm^{-3}$, how many excited atoms in the state 2 ($J_2 = 1$) are required to establish a small-signal gain coefficient of 5%/m?

given: $[N_2 - (g_2/g_1) N_1]\sigma = .05\,m^{-1} = 5 \cdot 10^{-4}\,cm^{-1}$

$(N_2 - (g_2/g_1) N_1) = 9.22 \cdot 10^8\,cm^{-3}$

(eq 7.37) $g_2 = 2J_2 + 1$ $g_1 = 2J_1 + 1$

$g_{2(1)}$ = number of ways an atom can have energy $E_{2(1)}$

$\therefore g_2/g_1 = 3/5$

$N_2 = 9.22 \cdot 10^8 + (3/5) N_1$ $N_1 = 10^{10}\,cm^{-3}$

$N_2 = (9.22 \cdot 10^8 + (\frac{3}{5}) \cdot 10^{10}) / cm^{3}$

$N_2 = 6.922 \cdot 10^9 / cm^3$

$N_1 > N_2$ and we have a gain.

9.5 Fig. 9.4 $R_1 = .95 \quad T_1 = 0$

$R_2 = .8 \quad T_2 = .2$

$l_g = 10cm \quad d = 15cm$

$\lambda_0 = 7200Å$

$\sigma = 10^{-18} cm^2 \quad I_s = 20kW/cm^2$

a) What is the photon lifetime?

$$\tau = \frac{2d/c}{1 - R_1/R_2} = 4.17 \cdot 10^{-9} s$$

b) What is the inversion density $[N_2 - (g_2/g_1)N_1]$ necessary to reach threshold for CW oscillation?

$R_1 R_2 \exp\{2\gamma_0 l_g\} = 1 \qquad \gamma =$

$$\gamma = \Delta N \sigma = \frac{1}{2l_g} \ln\left(\frac{1}{R_1 R_2}\right) = 1.37 \cdot 10^{-2} \, cm^{-1}$$

$$\Delta N = \frac{1.37 \cdot 10^{-2} cm^{-1}}{10^{-18} cm^2} = 1.37 \cdot 10^{16} cm^{-3}$$

c) Suppose the cavity mode is "seeded" with a running wave of 10^8 photons/cm² at $\lambda = 7200Å$ at $t=0$, then the inversion of $2 \cdot 10^{17} cm^{-3}$ is created instantaneously (by external pumps). Estimate the time interval req'd for the internal intensity of the laser to reach half the Saturation value.

$I < I_s \qquad G_0 = \exp(\gamma_0 l_g) = 7.39$ for $\Delta N = 2 \cdot 10^{17} cm^{-3}$

Open the cavity and follow the photons

$$N_p(z) = N_p(0) \exp^{[\gamma_0 - \alpha] z}$$

$\gamma_0 = 0.2 cm^{-1} \qquad \alpha = 1.37 \cdot 10^{-2} cm^{-1}$

$I_s = kW/cm^3 = h\nu \, c \, N_{ps}$

$N_{ps} = 2.41 \cdot 10^{12}$ photons/cm²

$$(\gamma_0 - \alpha) l_g = \ln\left\{\frac{(1/2)\, 2.41 \cdot 10^{12}}{10^8}\right\} = 9.4$$

$\gamma_0 = 2 \cdot 10^{17} cm^{-3} \cdot 10^{-18} cm^2 = .2 cm^{-1}$

$\alpha = \frac{1}{2l_g} \ln\left(\frac{1}{R_1 R_2}\right) = 1.37 \cdot 10^{-2} cm^{-1} \quad (\gamma_0 - \alpha) = 1.86 \cdot 10^{-1} cm^{-1}$

$\therefore z = 50.46 cm$

$\therefore \Delta t = \left(\frac{z}{c}\right) = 1.68 \cdot 10^{-9} s$

actual time is longer, I_s was ignored

9.9 Ring Laser

$R_1 = .96 \quad R_2 = .8 \quad R_3 = .97 \quad R_4 = .98$

$T_1 = T_3 = T_4 = 0 \quad T_2 = .2 \quad \lambda = 760\,nm$

$E_2 = 3.2\,eV \quad \sigma = 2 \cdot 10^{-20}\,cm^2$

$\tau_2 = 1.54\,ms$ pump rate $= R_{02}$

$\tau_1 \sim 0 \quad N_1 = 0$

a) find the upper-state density required to reach threshold, assuming steady-state

$$R_1 R_2 R_3 R_4 T_a T_b \exp[\gamma_{th} l_g] = 1$$

$$\gamma_{th} = N_2 \sigma = 1.573 \cdot 10^{-7}\,cm^{-1}$$

$$N_{2th} = 7.36 \cdot 10^{18}\,cm^{-3}$$

b) $$\frac{dN_2}{dt} = R_{02} - \frac{N_2}{\tau_2} = 0$$

$$\Rightarrow R_{02} = \frac{N_2}{\tau_2} = 5.1 \cdot 10^{20}\,cm^{-3}\,s^{-1}$$

$$\text{Power/Volume} = (3.2)(1.6 \cdot 10^{-19} R_{02})$$

$$= 262\,W/cm^3$$

c) $$\frac{dN_2}{dt} = R_2 - N_2\left[\frac{1}{\tau_2} - \frac{\sigma I}{h\nu}\right] = 0$$

$$N_2 = 5 R_{02} \tau_2$$

$$N_{2th}\left(\frac{\sigma I}{h\nu}\right) = R_2 - R_{20} = .5 R_{20} = \frac{N_{2th}}{\tau_2}$$

$$I = \frac{h\nu}{\sigma}\left[\frac{R_2 - R_{20}}{N_{2th}}\right] \sim 4.23 \cdot 10^3\,kW/cm^2$$

$$I_{out} = 0.2 I = 847\,W/cm^2$$

(eq 9.2.6)

9.12 $E_0 = 2eV$ $\nu_0 = 1.5GHz$

$\sigma = 10^{-16}\ cm^2$ $\tau_2 = 1ms$

branching ratio $= 0.8$

$\tau_1 = 10\mu s$ equal degeneracy

Cavity of 9.4 $R_1 = .98$ $(T_1 = 0)$

$R_2 = .9$ $(T_2 = .1)$

$l_g = 30cm$ cavity $d = 50cm$

a) $\gamma_{th} = \frac{1}{2l_g}\ln\left(\frac{1}{R_1R_2}\right) = 1.756 \cdot 10^{-3}\ cm^{-1}$

$N_2 - \frac{g_2}{g_1}N_1 = 1.756 \cdot 10^{13}\ cm^{-3}$

$I_s = \frac{h\nu}{\sigma\tau_2} = 3.2\ W/cm^2$

b) Laser oscillates at the cavity that has the highest gain to loss ratio.

c) $\frac{I^+}{I_s} = \frac{1}{2}\left[\frac{\gamma_0}{\alpha} - 1\right] = 0.25$ $I^+ = .8\,W/cm^2$

d) $\frac{I^+}{I_s} = \frac{\gamma_0 l_g - \frac{1}{2}\ln(1/R_1R_2)}{(1-\sqrt{R_1R_2})(1+\sqrt{R_1R_2})} = 0.221$

$I^+ = 0.7073\ W/cm^2$ $\left(\gamma_0 l_g = 1.5\left(\frac{1}{2}\right)\ln\left(\frac{1}{R_1R_2}\right)\right)$