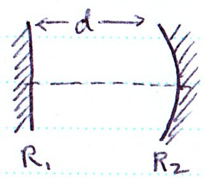
6.1 The following questions refer to the optical cavity shown in Fig. 6.5 with $d = (3/4)R_2$ $\Gamma_1^2 = 0.99$ and $\Gamma_2^2 = 0.97$



a) Find an expression for the resonant frequencies of the $TEM_{0,0}$ modes of the cavity.

$$\nu_{m,p,q} = \frac{c}{2nd}\left\{q + \left(\frac{1+m+p}{\pi}\right)\tan^{-1}\left[\frac{(d/R_2)^{1/2}}{(1-d/R_2)^{1/2}}\right]\right\}$$

$m=0$ $p=0$ $d=\frac{3}{4}R_2$

$$\nu_{0,0,q} = \frac{c}{2nd}\left\{q + \left(\frac{1}{\pi}\right)\tan^{-1}\left[\frac{(3/4)^{1/2}}{(1-3/4)^{1/2}}\right]\right\}$$

$$\nu_{0,0,q} = \frac{c}{2nd}\left\{q + \left(\frac{1}{\pi}\right)\tan^{-1}\sqrt{3}\right\}$$

b) If the radius of curvature is 2.0 m and the wavelength region of interest 5000 Å, compute:

(1) Free spectral range in MHz and in Å units

$R_2 = \frac{4}{3}d \Rightarrow d = \frac{3}{4}R_2 = \frac{3}{2}m = 1.5m$ $\quad n = 1.0^+$

$$F.S.R. = \nu_{N+1} - \nu_N = \frac{c}{2nd} = \frac{3\cdot 10^8 ms^{-1}}{2(1.5m)} = 10^8 Hz$$

$\lambda = c/\nu$ $\quad$ ~~$d\lambda$~~ $\nu = c/\lambda$ $\quad d\nu = (c/\lambda^2)|d\lambda|$

$$|d\lambda| = \frac{\lambda^2 d\nu}{c} = \frac{\lambda^2(FSR)}{c} = \frac{(5000\cdot 10^{-10}m)^2 10^8 s^{-1}}{3\cdot 10^8 ms^{-1}} = 8.\bar{3}\cdot 10^{-14} m$$

$$|d\lambda| = 8.\bar{3}\cdot 10^{-4} Å$$

(2) Cavity Q

reflectance $R_1 = |\Gamma_1|^2 = .99$ $\quad R_2 = |\Gamma_2^2| = 0.97$ (not radius of curvature)

$$Q = \frac{2\pi nd}{\lambda_0}\frac{(R_1R_2)^{1/4}}{(1-\sqrt{R_1R_2})} = \frac{2\pi(\overset{1.5}{\cancel{2}}m)}{5000\cdot 10^{-10}m}\frac{(.99\cdot .97)^{1/4}}{[1-(.99\cdot .97)^{1/2}]}$$

$$Q = 930607069.741143$$

(3) Photon lifetime in seconds

$\omega_0 = 2\pi c/\lambda_0$

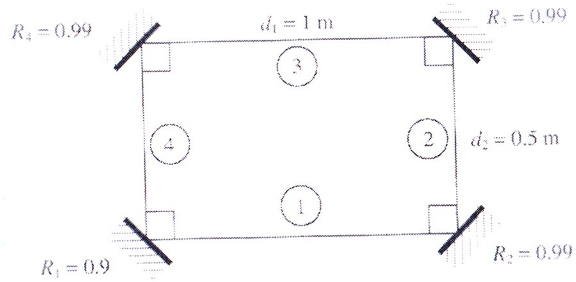
$$\tau_p = Q/\omega_0 = Q\lambda_0/2\pi c = \frac{2\pi nd}{\lambda_0}\frac{(R_1R_2)^{1/4}}{(1-\sqrt{R_1R_2})}\frac{\lambda_0}{2\pi c}$$

$$\tau_p = \frac{nd}{c}\frac{(R_1R_2)^{1/4}}{(1-\sqrt{R_1R_2})} = \frac{1.5}{3\cdot 10^8}\frac{(.99\cdot .97)^{1/4}}{[1-(.99\cdot .97)^{1/2}]} = 2.47\cdot 10^{-7}s$$

(4) Finesse

$$F = \pi\frac{(R_1R_2)^{1/4}}{[1-\sqrt{R_1R_2}]} = \cancel{\frac{2\pi}{1-\sqrt{R_1R_2}}} \cancel{=} \cancel{2.468812}$$

$$F = 155.101$$



6.2 If the optical paths 1-4 are lossless, what is the photon lifetime in this cavity?

$S = R_1 R_2 R_3 R_4 = (.99)^3(.9) = 0.873269$ = Survival of photon in one Round trip

$\tau_{RT} = \frac{\Sigma d_i}{c} = \frac{(1+.5+1+.5)m}{3 \cdot 10^8 m/s} = \frac{3\,s}{3 \cdot 10^8} = 10^{-8} s$

$\tau_p = \frac{\tau_{RT}}{(1-S)} = \frac{10^{-8}}{(1-.8732)} = 7.89 \cdot 10^{-8} s = 78.9\,ns$

6.3 What is the cavity Q ($\lambda_0 = 5000 Å$)

$Q = \omega_0 \tau_p = \left(\frac{2\pi c}{\lambda_0}\right)\tau_p = \frac{2\pi(3 \cdot 10^8 ms^{-1})(7.89 \cdot 10^{-8} s)}{5000 \cdot 10^{-10} m}$

$Q = 297445992.4 = 2.97 \cdot 10^8$

6.4 a) Suppose that path 1 has a transmission coefficient of 0.85 rather than 1 as in Problem 6.2. What is the new photon lifetime?

$S = R_1 T_1 R_2 R_3 R_4 = (.99)(.85)(.99)^2(.9) = 0.742$

$\tau_{RT} = 10^{-8} s$

$\tau_p = \frac{\tau_{RT}}{(1-S)} = \frac{10^{-8}}{(1-.742)} = 3.88 \cdot 10^{-8} s = 38.8\,ns$

b) Suppose path 1 has a ~~transmission coefficient~~ power gain of 1.1. What is the new photon ~~lifetime~~?

$S = R_1 G R_2 R_3 R_4 = (.99)^3(.9)(1.1) = 0.960596$

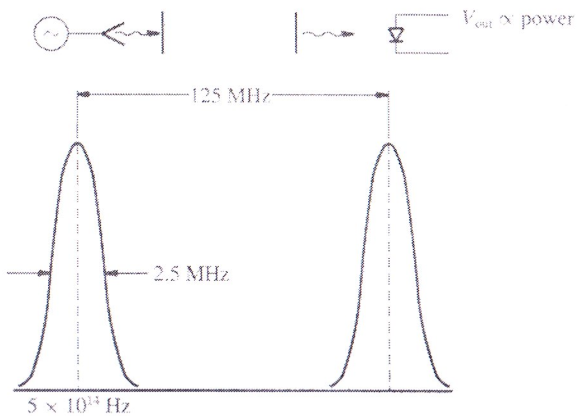
$\tau_{RT} = 10^{-8} s$

$\tau_p = \frac{\tau_{RT}}{(1-S)} = \frac{10^{-8}}{(1-.960596)} = 25.4 \cdot 10^{-8} s = 254\,ns$

c) If we blindly plug into the formulas τ_p becomes negative for G sufficiently large. What is the meaning of this absurdity?

means infinite output with zero line-width.

Problems 6.5 through 6.10 refer to the optical cavity in the accompanying diagram. It is excited by a variable-frequency source, and the detected intensity is as shown.



6.5 What is the nominal wavelength of the source?

$$\lambda_0 = \frac{c}{\nu_0} = \frac{3 \cdot 10^8 ms^{-1}}{5 \cdot 10^{14} Hz} = 6 \cdot 10^{-7} m = 0.6 \mu m$$

6.6 How long is the cavity?

$$FSR = \frac{c}{2d} = \Delta\nu \Rightarrow d = \frac{c}{2\Delta\nu} = \frac{3 \cdot 10^8 ms^{-1}}{2(125 \cdot 10^6 Hz)} = 1.2 m$$

6.7 What is the Finesse?

$$F = \frac{FSR}{\Delta\nu_{1/2}} = \frac{125 MHz}{2.5 MHz} = 50$$

6.8 What is the Q?

$$Q = \frac{\nu_0}{\Delta\nu_{1/2}} = \frac{5 \cdot 10^{14} Hz}{2.5 \cdot 10^6 Hz} = 2 \cdot 10^8$$

6.9 What is the photon lifetime?

$$\tau_p = \frac{1}{\Delta\omega_{1/2}} = \frac{1}{2\pi\Delta\nu_{1/2}} = \frac{1}{2\pi(2.5 \cdot 10^6 Hz)} = 6.37 \cdot 10^{-8} s$$

6.10 Suppose that the cavity is filled with an active medium with a single-pass gain of G. How large should G be to obtain oscillation?

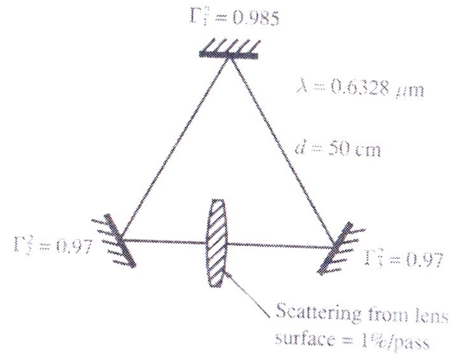
$$\tau_p = \frac{2d/c}{1 - R_1R_2} = 63.7 ns \Rightarrow$$

$$1 - R_1R_2 = {}^{2d}/_{c\tau_p} \Rightarrow R_1R_2 = 1 - {}^{2d}/_{c\tau_p} = 0.8743$$

The condition for oscillation is that τ_p is a negative value.

$$1 - G^2R_1R_2 < 0$$

$$G > \frac{1}{\sqrt{R_1R_2}} = \frac{1}{\sqrt{1 - {}^{2d}/_{c\tau_p}}} = 1.069$$



6.11a) What is the photon lifetime?

$$S = T_{lens}\,\Gamma_3^2\,\Gamma_1^2\,\Gamma_2^2 = (1.01)(.97)(.985)(.97) = \cancel{0.91752}\ .936$$

$$d = \Sigma d_n = 3 \cdot 50\,cm = 1.5m \Rightarrow \tau_{RT} = \frac{d}{c} = \frac{1.5m}{3 \cdot 10^8 m/s} = 5 \cdot 10^{-9} s$$

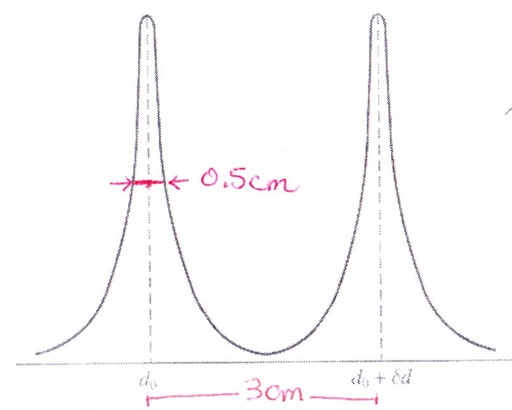
$$\tau_p = \frac{\tau_{RT}}{(1-S)} = \frac{5 \cdot 10^{-9} s}{(1 - .936)} = 78.66 ns$$

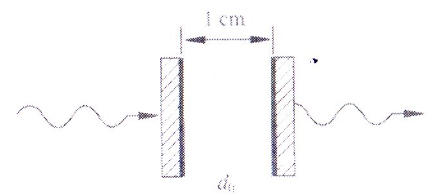
b) What is the cavity Q?

$$Q = \omega_0 \tau_p = \frac{2\pi c\,\tau_p}{\lambda_0} = \frac{2\pi(3 \cdot 10^8)(78.66 \cdot 10^{-9} s)}{.6328 \cdot 10^{-6} m}$$

$$Q = 234308797.2 = 2.34 \cdot 10^8$$

Drawn to scale on the graph below is the relative power transmission through a Fabry-Perot cavity when the distance d is increased slightly. The source is a He:Ne laser at $\lambda_0 = 6328$ Å.





6.12a) What is the distance δd?

$$\lambda m = 2dn_f \cos\theta + \phi\lambda_0/\pi \qquad \cos\theta = 1$$

$$[(m+1) - m]\lambda_0 = (d + \delta d)\,2n_f - d(2n_f)$$

$$\Rightarrow \delta d = \lambda_0/2n_f = \lambda_0/2 = 3164\text{ Å} = 3164 \cdot 10^{-7} m$$

b) What is the finesse of the cavity?

τ_p from chart $3cm = \lambda_0/2 = 3164\text{ Å} = FSR = 2.4 \cdot 10^{14} Hz$

$\therefore\ \Delta\lambda_{1/2} = .5\,cm = 5.3 \cdot 10^{-8} m$

$$\nu = c/\lambda \quad \Delta\nu_{1/2} = c\Delta\lambda/\lambda_0^2 = (3 \cdot 10^8 ms^{-1})(5.3 \cdot 10^{-8} m)/(6328 \cdot 10^{-10} m)^2$$

$$\Delta\nu_{1/2} = 3.97 \cdot 10^{13} Hz$$

$$F = \frac{FSR}{FWHM} = \frac{\Delta\nu}{\Delta\nu_{1/2}} = \frac{\Delta\lambda}{\Delta\lambda_{1/2}} = \frac{3164\text{ Å}}{5.3 \cdot 10^{-8} m} = 5.97$$

c) What is the cavity Q?

$$Q = \frac{\lambda_0}{\Delta\lambda_{1/2}} = \frac{6328\text{ Å}}{5.3 \cdot 10^{-8} m} = 11.9$$

→

6.12 a) What is the distance δd? $d_0 = 1\,cm$ $n=1$

$FSR = c/2nd = 3\cdot10^8 m/s / (2\cdot10^{-2} m)$

$FSR = 1.5\cdot10^{10}\ Hz$

b) What is the finesse of the cavity?

$\Delta\nu_{1/2} \sim \frac{1}{6} FSR = FWHM$

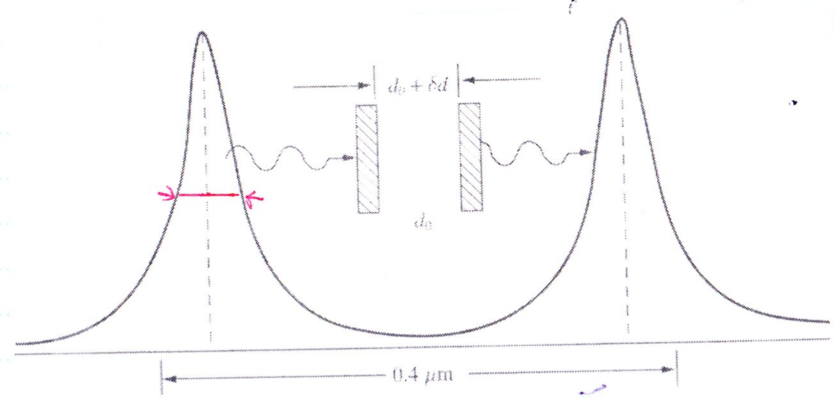
$$F = \frac{FSR}{FWHM} \approx \frac{FSR}{\frac{1}{6}FSR} \approx 6$$

c) What is the cavity Q?

$2N\pi = (4\pi/\lambda_0)\, nd\cos\theta + \delta_r = \Delta \Rightarrow N = \frac{2}{\lambda_0}d = 31606$

$$Q = \frac{\lambda_0}{\Delta\lambda_{1/2}} = \frac{\nu_0}{\Delta\nu_{1/2}} = N\mathcal{F} = \frac{N\pi\sqrt{F}}{2} = \frac{31606\pi\sqrt{6}}{2}$$

$$= 121607$$



a) What is the wavelength of the source?

$2N\pi = 4\pi/\lambda_0\, nd$

$2(N+1)\pi = 4\pi/\lambda_0\, n(d+\delta d)$ $\delta d \sim 0.25\,\mu m$

$\lambda_0 = (2/n)(\delta d) = 0.5\,\mu m$

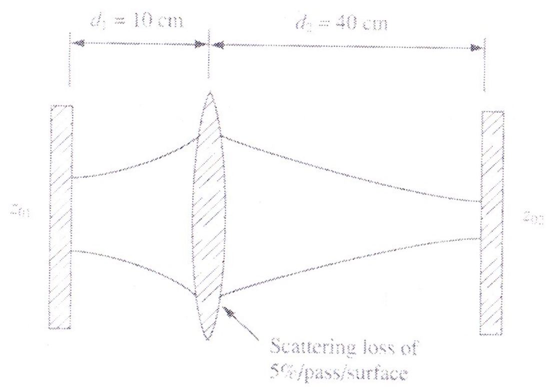
b) What is the finesse?

$FSR = c/2nd = (3\cdot10^8 ms^{-1})/2(2\cdot10^{-2}m) = 7.5\cdot10^9\ Hz$

$FWHM \sim \frac{1}{5.5}FSR \Rightarrow F \sim 5.5$

(a) Find an expression for the difference in the resonant frequencies of the $TEM_{0,0,q}$ and the $TEM_{m,n,q}$ modes of the cavity shown below. You may assume that the cavity is stable and the parameters z_{01} and z_{02} are known. Express your answer in terms of d_1, d_2, z_{01}, and z_{02} —(do not evaluate).

(b) What is the photon lifetime and Q of the passive cavity if $\lambda_0 = 500$ nm and $R_1 = R_2 = 0.876$? (A numerical answer is required.)



6.16 a) Expressions for $TEM_{0,0,q}$ and $TEM_{m,n,q}$.

using 3.5.1 $\phi(d) - \phi(0) = kd - (1+m+p)\tan^{-1}(d/z_0)$

$kd_1 - (1+m+p)\tan^{-1}(\frac{d_1}{z_0}) + kd_2 - (1+m+p)\tan^{-1}(\frac{d_2}{z_0}) = q\pi$

$k(d_1+d_2) = \pi\{q + \frac{(1+m+p)}{\pi}[\tan^{-1}d_1/z_0 + \tan^{-1}d_2/z_0]\}$

$k = \frac{2\pi\nu n}{c}$ n = index of refraction in cavity

$\nu_{m,p,q} = \frac{cn}{(d_1+d_2)}\{q + \frac{(1+m+p)}{\pi}[\tan^{-1}d_1/z_0 + \tan^{-1}d_2/z_0]\}$

$\nu_{0,0,q} = \frac{cn}{(d_1+d_2)}\{q + \frac{1}{\pi}[\tan^{-1}d_1/z_0 + \tan^{-1}d_2/z_0]\}$

b) τ_p, Q given $R_1 = R_2 = 0.876$ $T = 1.05$
$\lambda_0 = 500$ nm

$\tau_{RT} = \frac{\Sigma d}{c} = \frac{2(10\text{cm} + 40\text{cm})}{3\cdot 10^8 \text{m/s}} = 3.3\bar{3}\cdot 10^{-9}$ s

$S = R_1 \underbrace{TTTT}R_2 = (\cancel{1.05}\,^{(.95)})^4(.876)^2 = \cancel{0.93275}\ ^{.625}$

two passes, two surfaces scattering loss!

$\tau_p = \frac{\tau_{RT}}{(1-S)} = \cancel{49.5\text{ns}}$ 10.8ns

$\Delta\nu_{1/2} = \frac{1}{2\pi\tau_p} = \cancel{3.214\cdot 10^6\text{Hz}}$ $1.47\cdot 10^7$ Hz

$\nu_0 = \frac{c}{\lambda_0} = 6\cdot 10^{14}$ Hz

$Q = \frac{\nu_0}{\Delta\nu_{1/2}} = \cancel{1.87\cdot 10^8}$ $4.09\cdot 10^7$

6.17 Make a careful sketch of the peaks (and valleys) of the transmission through a Fabry-Perot cavity as a function of frequency around $\lambda_0 = 6000Å$ with a finesse of 10 and FSR = 20GHz

a) $FWHM = \frac{FSR}{F} = \frac{20\,GHz}{10} = 2GHz$

$\nu = c/\lambda \Rightarrow \Delta\nu_{1/2} = (c/\lambda_0^2)\Delta\lambda_{1/2}$

$\Delta\lambda_{1/2} = \lambda_0^2 \Delta\nu_{1/2}\left(\frac{1}{c}\right) = (6000Å)^2(2GHz)\left(\frac{1}{3\cdot10^8 m/s}\right)$

$\Delta\lambda_{1/2} = 0{,}024\,Å$

$\Delta k_{1/2} = \frac{2\pi}{\Delta\lambda_{1/2}} = 2.62\cdot10^{12}\,cm^{-1}$

b) What is the cavity Q?

$Q = \frac{\lambda_0}{\Delta\lambda_{1/2}} = \frac{6000Å}{.024Å} = 250{,}000$

c) What is the photon lifetime?

$\tau_p = \frac{Q}{\omega_0} = \frac{Q\lambda_0}{2\pi c} = \frac{(250{,}000)(6000\cdot10^{-10}m)}{2\pi(3\cdot10^8 ms^{-1})} = 7.96\cdot10^{-11}s$

d) What is the FSR in GHz, Å, and cm^{-1}?

FSR = 20GHz

$FSR = F\cdot FWHM = 10\,\Delta\lambda_{1/2} = 0.24Å$

$FSR = F\cdot FWHM = 10\,\Delta k_{1/2} = 26\cdot10^{12}\,cm^{-1}$

transmission $\theta = kd$ $\quad d = \frac{c}{2FSR} = \frac{3\cdot10^8 ms^{-1}}{2(20\cdot10^9 s^{-1})} = 0.0075m = 7.5mm$

$\theta = \left(\frac{2\pi\nu}{c}\right)d \quad k = \frac{\omega}{c} = \frac{2\pi\nu}{c}$

$\theta = \frac{2\pi\nu}{c}\left(\frac{c}{2FSR}\right) = \frac{\pi\nu}{FSR}$

$F = \frac{4R}{(1-R)^2} \Rightarrow (1-2R+R^2)F = 4R$

$FR^2 - (F2+4)R + F = 0$

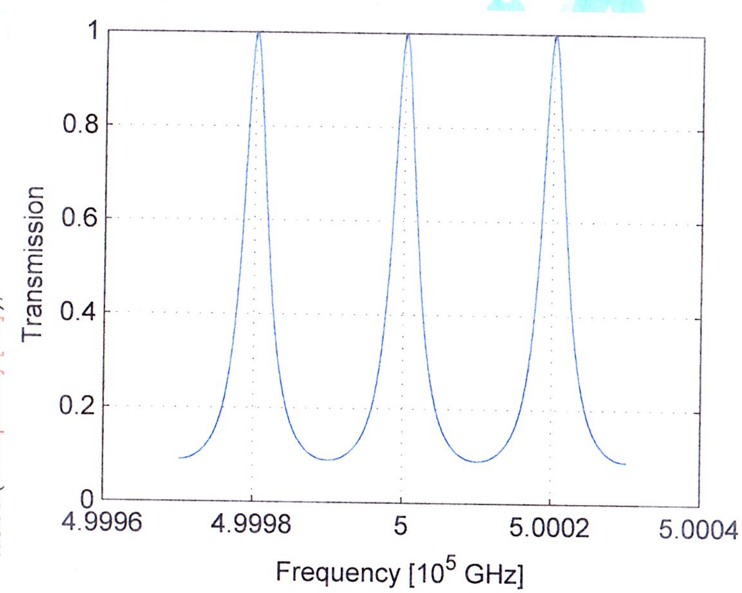
$R^2 - \left(\frac{F2+4}{F}\right)R + 1 = 0$

$F = 10 \quad R = \frac{\left(\frac{2F+4}{F}\right) \pm \sqrt{\left(\frac{F2+4}{F}\right)^2 - 4}}{2}$

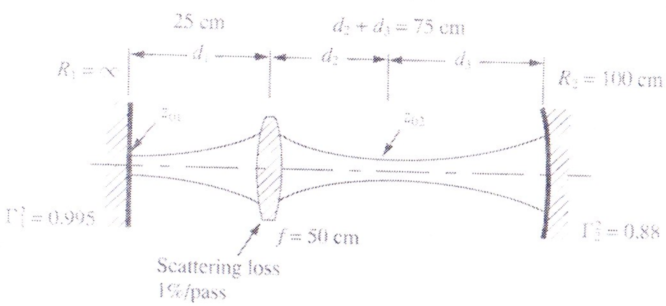
$R = 1.2 \pm .66 = .54$

$T(\theta) = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2\theta} = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2\left(\frac{\pi\nu}{FSR}\right)}$

```
function Lasers_Ch6_Pr17
Lambda0=6000e-10;FSR=20e9;c=3e8;
v0=c/Lambda0;R=0.54;R2=(1-R)^2;dv=2e9;
v=v0-3*FSR/2:dv/20:v0+3*FSR/2;theta=pi*v/FSR;
T=R2./(R2+4*R*(sin(theta)).*(sin(theta)));
plot(v*1e-9,T,'b'),grid on;hold on;
ylabel('Transmission');
xlabel('Frequency [Hz]');
```



6.18. Consider the following laser cavity and assume that the parameters z_{01} and z_{02} are known. Assume also that the cavity is stable.



a) What is the radius of curvature of the phase at the spherical mirror?

force the phase surface to match the curved mirror at d_3: $R(3) = R_2 = 100\text{cm}$

b) What is the photon lifetime?

$$\tau_{RT} = \frac{2\sum d_i}{c} = \frac{2(100\text{cm})}{3\cdot 10^{10}\text{cm s}^{-1}} = 6.7\cdot 10^{-9}\text{ s}$$

$$S = (T_{Loss})^2 \Gamma_1^2 \Gamma_2^2 = (.99)^2(.995)(.88) = 0.858$$

$$\tau_p = \frac{\tau_{RT}}{(1-S)} = 4.7\cdot 10^{-8}\text{s} = 47\text{ ns}$$

c) Derive a formula for the resonant frequency of the $TEM_{m,p,q}$ mode.

use 3.5.1 $\phi(d) - \phi(0) = kd - (1+m+p)\tan^{-1}(d/z_0) = q\pi$
and $k = 2\pi\nu n/c$ n = index of refraction

$$kd_1 - (1+m+p)\tan^{-1}\left(\frac{d_1}{z_{01}}\right) + kd_2 - (1+m+p)\tan^{-1}\left(\frac{d_2}{z_{02}}\right)$$
$$+ kd_3 - (1+m+p)\tan^{-1}\left(\frac{d_3}{z_{03}}\right) = q\pi$$

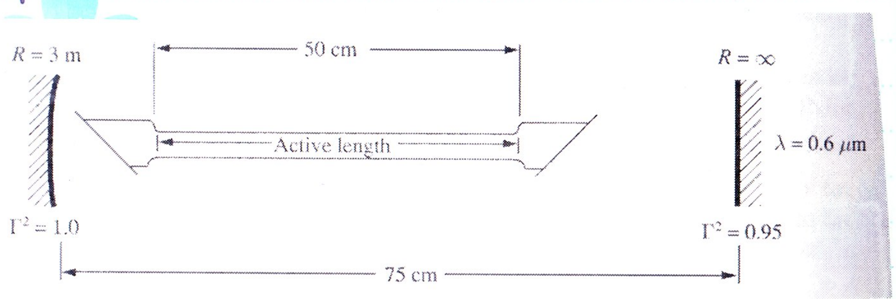
$$k(d_1+d_2+d_3) = q\pi + (1+m+p)\left\{\tan^{-1}\frac{d_1}{z_{01}} + \tan^{-1}\frac{d_2}{z_{02}} + \tan^{-1}\frac{d_3}{z_{03}}\right\}$$

$$k = \pi(d_1+d_2+d_3)^{-1}\left\{q + \frac{(1+m+p)}{\pi}\left[\tan^{-1}\frac{d_1}{z_{01}} + \tan^{-1}\frac{d_2}{z_{02}} + \tan^{-1}\frac{d_3}{z_{03}}\right]\right\}$$

$$\frac{2\pi\nu}{c} = \pi(d_1+d_2+d_3)^{-1}\left\{q + \frac{(1+m+p)}{\pi}\left[\tan^{-1}\frac{d_1}{z_{01}} + \tan^{-1}\frac{d_2}{z_{02}} + \tan^{-1}\frac{d_3}{z_{03}}\right]\right\}$$

$$\nu_{m,p,q} = \frac{c}{2(d_1+d_2+d_3)}\left\{q + \frac{(1+m+p)}{\pi}\left[\tan^{-1}\frac{d_1}{z_{01}} + \tan^{-1}\frac{d_2}{z_{02}} + \tan^{-1}\frac{d_3}{z_{03}}\right]\right\}$$

6.19



a) Is this cavity stable?

$g_1 = 1 - \frac{d}{R_1} = 1 - \frac{75cm}{300cm} = 0.75$ eq. 2.5.4

$g_2 = 1 - \frac{d}{R_2} = 1$ $g_1 g_2 = 0.75$

Stability condition: $0 \le g_1 g_2 \le 1$ ∴ Stable

b) What would be the frequency difference between the $TEM_{0,0,q}$ mode and the $TEM_{1,0,q}$ mode?

$\nu_{m,p,q} = \frac{c}{2d}\left\{q + \frac{(1+m+p)}{\pi}\cos^{-1}(1-\frac{d}{R})^{1/2}\right\}$

$\frac{c}{2d} = \frac{3\cdot 10^{10} cm\, s^{-1}}{2\,(75cm)} = 2\cdot 10^8\, Hz$

$\cos^{-1}\sqrt{\frac{3}{4}} = \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$

$\nu_{m,0,q} = 2\cdot 10^8\, Hz\left\{q + \frac{(1+m)}{\pi}\frac{\pi}{6}\right\}$

$\nu_{1,0,q} - \nu_{0,0,q} = 2\cdot 10^8\, Hz\left(\frac{1}{6}\right) = 3.3\bar{3}\cdot 10^7\, Hz = 33.3 MHz$

$\Delta\nu = 33.3 MHz$

c) What should be the bore size of the laser tube so that less than 0.1% of the $TEM_{0,0,q}$ mode intercepts the tube wall?

$T = 1 - e^{-2a^2/w_s^2} = 0.999$

$-2a^2/w_s^2 = \ln(.001)$

$a = 1.858\, w_s$

$\frac{\pi w_s^2}{\lambda} = \frac{dR}{(1-d/R)^{1/2}} \Rightarrow w_s = \sqrt{\frac{(dR)^{1/2}\lambda}{\pi(1-d/R)^{1/2}}}$

$w_s = \sqrt{\frac{(75\cdot 10^{-2})(3))^{1/2}\cdot 6\cdot 10^{-6})}{\pi(1-75/300)^{1/2}}} = 5.75\cdot 10^{-4} m$

$a = (1.858)(5.75\cdot 10^{-4} m) = 1.07\cdot 10^{-3} m$

diameter $= 2a = 2.14\cdot 10^{-3} m = 2.14\, mm$

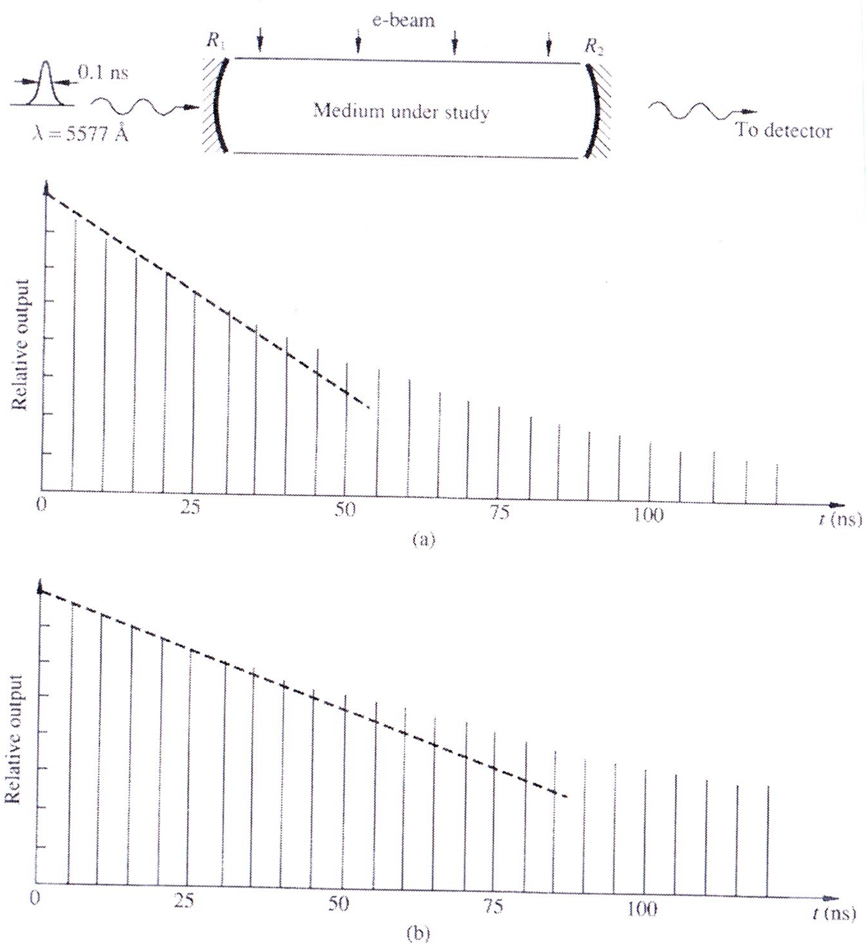
d) What is the minimum gain coefficient of the laser tube to sustain oscillation?

oscillator condition: $R_1 R_2 \exp\{\gamma - 2l_g\} = 1$

$\gamma = \frac{1}{2l_g}\ln\left(\frac{1}{R_1R_2}\right) = \frac{1}{2l_g}\ln\left(\frac{1}{\Gamma_1^2\Gamma_2^2}\right) = \frac{1}{2(50cm)}\ln\left(\frac{1}{.95}\right) =$

$5.129\cdot 10^{-4} cm^{-1}$

o.20 A laser cavity was excited by a 0.1ns pulse from an external source at $\lambda = 5577\text{Å}$. When the medium was not pumped, the detected transmission was as shown in the part (a) of the diagram. When the medium was irradiated by an intense electron beam, part (b) of the diagram resulted.



a) How long is the cavity?

$2d = c\tau_{RT}$ $\qquad \tau_{RT} = 5ns$

$d = \frac{c\tau_{RT}}{2} = \frac{(3\cdot 10^8 ms^{-1})(5\cdot 10^{-9}s)}{2} \doteq 0.75\,m = 75cm$

b) What is the photon lifetime?

From plot - relative units $I_{max} = 8$

at τ_p $\quad I_{max} = 8/e \approx 2.94$

$\therefore \tau_p \sim 75ns$

c) What is the cavity Q?

$$Q = \omega_0 \tau_p = \frac{2\pi c \tau_p}{\lambda_0} = \frac{2\pi(3\cdot 10^8 ms^{-1})(75\cdot 10^{-9}s)}{(5577\cdot 10^{-10}m)}$$

$Q = 2.5349 \cdot 10^8$

d) What is the single pass gain under e-beam excitation?

$\tau_p = \frac{\tau_{RT}}{1-S}$ $\qquad S = R_1R_2 \Rightarrow S = \gamma^2 R_1R_2$

$R_1R_2 = 1 - \frac{\tau_{RT}}{\tau_p} = 1 - \frac{5ns}{75ns} = .933$

$\gamma^2 R_1R_2 = 1 - \tau_{RT}/\tau_p' \Rightarrow \gamma = \sqrt{\frac{(1-\tau_{RT}/\tau_p')}{R_1R_2}}$

$\gamma = \sqrt{\frac{1 - \frac{5ns}{120ns}}{.933}} \sim 1.014$

e) What is the cold cavity finesse? $\Delta\omega_{1/2} = \frac{1}{\tau_p} \Rightarrow \Delta\nu_{1/2} = \frac{1}{2\pi\tau_p}$

$F = \frac{2\pi}{1-R_1R_2} = \frac{2\pi}{1-.933} \approx 94$

6.21 a) Is the cavity stable?

$$g_1 = 1 - d/R_1 = 1 - .75/1 = .25$$

$$g_2 = 1 - d/R_2 = 1 - .75/\infty = 1$$

$$0 \leq g_1 g_2 = .25 \leq 1 \quad \therefore \text{ stable}$$

b) What is the spot size of the beam at flat mirror?

$$w_0 = \left[\frac{\lambda}{\pi} (dR_2)^{1/2} \left(1 - \frac{d}{R_2}\right)^{1/2}\right]^{1/2} \qquad (5.2.2)$$