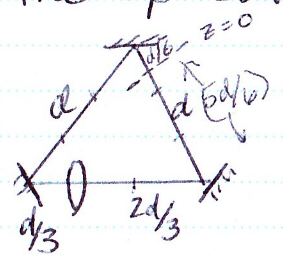
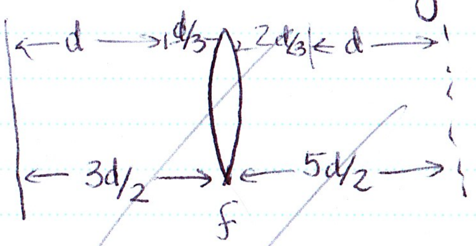
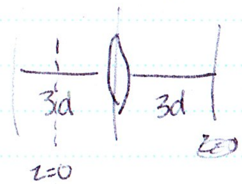
5.1 Consider the optical cavity (stable)



equivalent lens-waveguide like problem 2 →





$$T = \begin{bmatrix} 1 & 3d/2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & 5d/2 \\ 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 3d/2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5d/2 \\ -1/f & -5d/2f + 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 - 3d/2f & (5d/2 - 8d^2/4f + 3d/2) \\ -1/f & 5d/2 - 5d/2f + 1 \end{bmatrix}$$

~~min @ 3d/2~~

$$T = \begin{bmatrix} 1 & 3d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} = \begin{bmatrix} 1 - 3d/f & 3d \\ -1/f & 1 \end{bmatrix}$$

min @ $3d/2$

$$\frac{1}{q(z)} = \frac{A-D}{2B} - j\frac{\left[4 - \left(\frac{A+D}{2}\right)^2\right]^{1/2}}{B}$$

$$\frac{1}{q(z)} = \frac{-3d/f}{2(3d)} - j\frac{\left(1 - (1 - 3d/2f)^2\right)^{1/2}}{3d}$$

$$\frac{1}{q(z)} = -\frac{1}{2f} - j\left[\frac{\left[1 - (1 - 3d/2f)^2\right]^{1/2}}{3d}\right]$$

$$\frac{\pi n w^2(z)}{\lambda} = \frac{B}{\left[1 + \left(\frac{A+D}{2}\right)^2\right]^{1/2}}$$

$$\frac{\pi n w^2(z)}{\lambda} = \frac{3d}{\left[1 + (1 - 3d/2f)^2\right]^{1/2}}$$

Condition $(1 - 3d/2f) \geq 0$

$$\frac{3d}{2f} < 1$$

$$d/f < 2/3$$

minimum spot size

$$w_0 = \frac{\lambda}{\pi} \frac{B}{\left[1 + \left(\frac{A+D}{2}\right)^2\right]^{1/2}} = \frac{\lambda}{\pi} \frac{3d}{\left[1 + (1 - 3d/2f)^2\right]^{1/2}}$$

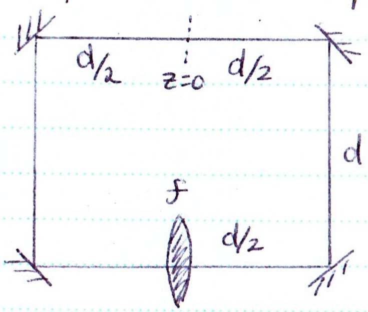
5.2 Find the spot size and the radius of curvature on the lens for the cavity shown below. The following procedure must be followed:

a) Show an equivalent waveguide with a unit cell starting just after the lens and proceeding in a counterclockwise fashion.

b) For what values of d/f is this cavity stable?

c) If $d = 20cm$, $f = 40cm$, $\lambda_0 = 6000Å$, find R & w (@ lens)

d) Identify the plane where the spot size is minim



a) —4d—— unit cell

b) $\begin{bmatrix} 1 & 4d \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} = \begin{bmatrix} 1-4d/f & 4d \\ -1/f & 1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$

Stable if $0 \le \frac{A+D+2}{4} \le 1$

$\frac{A+D+2}{4} = \frac{1-4d/f+1+2}{4} = 1 - d/f$

$0 \le 1 - d/f \le 1$ $\quad -1 \le \frac{A+D}{2} \le 1 \Rightarrow -1 \le 1 - 2d/f \le 1$

$\boxed{0 \le d \le f}$ $\quad d/f \le 1$

c) $d = 20cm$ $f = 40cm$ $\lambda_0 = 6000Å$ $z_0 @ 2d$
$z = 2d$

$R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right] = 4d\left[1 + \left(\frac{2d}{4d}\right)^2\right] = 5d = 100cm$

$W^2(z) = W_0^2\left[1 + \left(\frac{z}{z_0}\right)^2\right] = \frac{\lambda_0 z_0}{\pi n}\left[1 + \left(\frac{z}{z_0}\right)^2\right]$

$W^2(z) = \frac{\lambda(6000 \cdot 10^{-10} m)(2d)}{\pi(1)}\left[1 + \left(\frac{2d}{4d}\right)^2\right] = \frac{5\lambda_0 d}{\pi}$

$W^2(z) = \frac{(6000Å \cdot 10^{-10} m)5(20 \cdot 10^{-2} m)}{\pi} = 1.91 \cdot 10^{-7} m^2$

$w(z) = 4.4 \cdot 10^{-4} m$

d) $w(2d) = \frac{\lambda_0 z_0}{\pi n}\left[1 + \left(\frac{z}{z_0}\right)^2\right] = \lambda$

at lens
$R = 2f = 80cm$

$R(z) = z\left[1 + (z_0/z)^2\right]$

$R(2d) = 2d\left[1 + (z_0/2d)^2\right] = 2d + z_0^2/2d = R_2 = 2f$

$z_0^2 = (2f - 2d)2d = 4d(f-d) = (\pi W_0^2/\lambda)^2$

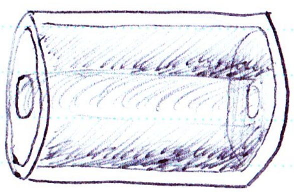
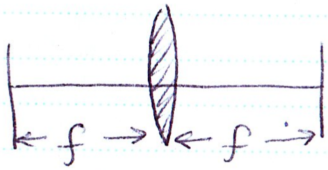
$W_0^2 = \frac{4\lambda[(f-d)d]^{1/2}}{\pi} = \frac{4 \cdot 6000 \cdot 10^{-8} cm[(40-20)20]^{1/2} cm}{\pi}$

$W_0^2 =$

5.3 The GRIN lens shown on the diagram below consists of a graded index fiber with $n(r) = n_0(1 - r^2/2a^2)$ of length $d = \pi a/2$ $a \gg r$

a) Find the ABCD matrix of this lens.

b) If we consider this short section of the fiber a cavity, what are the spot size at the entrance and exit planes?

 $\rightarrow$ 

$d = \pi a/2$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & n_0 \end{bmatrix} \begin{bmatrix} \cos d/a & a \sin d/a \\ -\frac{1}{a} \sin d/a & \cos d/a \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1/n_0 \end{bmatrix}$$

$$T = \begin{bmatrix} \cos d/a & a/n_0 \sin d/a \\ -n_0/a \sin d/a & \cos d/a \end{bmatrix} \xrightarrow[d = \frac{\pi a}{2}]{} \begin{bmatrix} 0 & a/n_0 \\ -n_0/a & 0 \end{bmatrix}$$

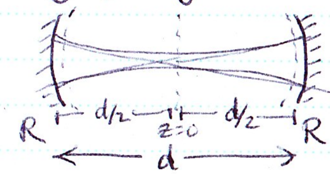
b) The matrix for round-trip

$$T = \begin{bmatrix} \cos 2d/a & a \sin 2d/a \\ -1/a \cos 2d/a & \cos 2d/a \end{bmatrix}$$

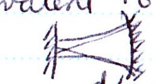
$$\frac{\pi W^2}{\lambda} = \frac{B}{[1 - (A+B)^2/4]^{1/2}} = \frac{a \sin(2d/a)}{[1 - \cos^2 2d/a]^{1/2}} = a$$

$$W = \sqrt{\frac{a\lambda}{\pi}}$$

5.4 Construct a graph similar to figure 5.5 for the cavity of Fig. 5.2 $(R_1 = R_2) = R$



symmetric actually equivalent to fig. 5.1 $\Rightarrow$



generally

$$R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right] \qquad W^2(z) = W_0^2\left[1 + \left(\frac{z}{z_0}\right)^2\right] \qquad z_0 = \frac{\pi n W_0^2}{\lambda_0}$$

force the phase surface to match the curved mirror @ $z = \frac{d}{2}$

$$R\left(\frac{d}{2}\right) = R = \frac{d}{2}\left[1 + \left(\frac{2z_0}{d}\right)^2\right] = \frac{d}{2}\left[1 + \frac{4z_0^2}{d^2}\right] = \frac{d}{2} + \frac{2z_0^2}{d}$$

$$z_0 = \sqrt{\left(R - \frac{d}{2}\right)\frac{d}{2}} = \sqrt{\frac{dR}{2}}\sqrt{1 - \frac{d}{2R}} = \frac{\pi W_0^2}{\lambda_0}$$

$$W_0^2 = \frac{\lambda_0 z_0}{\pi} = \frac{\lambda_0}{\pi}\sqrt{\frac{dR}{2}}\sqrt{1 - \frac{d}{2R}}$$

$$W^2(z) = \frac{\lambda_0 z_0}{\pi}\left[1 + \left(\frac{z}{z_0}\right)^2\right] = \frac{\lambda_0}{\pi}\sqrt{\frac{dR}{2}}\sqrt{1 - \frac{d}{2R}}\left[1 + \frac{z^2}{\left(\frac{dR}{2}\right)\left(1 - \frac{d}{2R}\right)}\right]$$

at mirror

$$W^2\left(\frac{d}{2}\right) = \frac{\lambda_0}{\pi}\sqrt{\frac{dR}{2}}\sqrt{1 - \frac{d}{2R}}\left[1 + \frac{d^2}{4\left(\frac{d}{2}\right)\left(1 - d/2R\right)}\right]$$

let $x = d/2R$

$$W^2\left(\frac{d}{2}\right) = \frac{\lambda_0}{\pi} R\sqrt{x}\sqrt{1 - x}\left[1 + \frac{\cancel{d^2}\,Rx}{\cancel{4}\,2(1-x)}\right]$$

at center

$$W^2(0) = \frac{\lambda_0}{\pi} z_0 = \frac{\lambda_0}{\pi}\sqrt{\frac{dR}{2}}\sqrt{1 - \frac{d}{2R}} = \frac{\lambda_0}{\pi} R\sqrt{x}\sqrt{1 - x}$$

$$W^2(z) = W_0^2\left[1 + \left(\frac{z}{z_0}\right)^2\right] \qquad z_0 = \frac{\pi n w_0^2}{\lambda_0}$$

$$W^2(z) = \frac{W_0^2}{z_0}\left[\frac{z_0^2 + z^2}{z_0}\right] \qquad \frac{W_0^2}{z_0} = \frac{\lambda_0}{\pi n}$$

$$W^2(z) = \frac{\lambda_0}{\pi}\left[\frac{z_0^2 + z^2}{z_0}\right]$$

$$z_0 = \sqrt{\frac{dR}{2}}\sqrt{1 - \frac{d}{2R}}$$

$$W^2(z) = \frac{\lambda_0}{\pi}\left[\frac{(dR/2)(1 - d/2R) + z^2}{\sqrt{\frac{dR}{2}}\sqrt{1 - d/2R}}\right]$$

@ $d/2$

$$W^2\left(\frac{d}{2}\right) = \frac{\lambda_0}{\pi}\,\frac{(dR/2)(1 - d/2R) + d^2/4}{\sqrt{\frac{dR}{2}}\sqrt{1 - d/2R}}$$

$$W^2\left(\frac{d}{2}\right) = \frac{\lambda_0}{\pi}\,\frac{dR/2 - d^2/4 + d^2/4}{\sqrt{\frac{dR}{2}}\sqrt{1 - d/2R}}$$

$$\cancel{W^2\left(\frac{d}{2}\right) = \frac{\lambda_0}{\pi}\frac{dR/2}{\sqrt{dR/2}}\left(1 - d/2R + d/4R\right)}$$

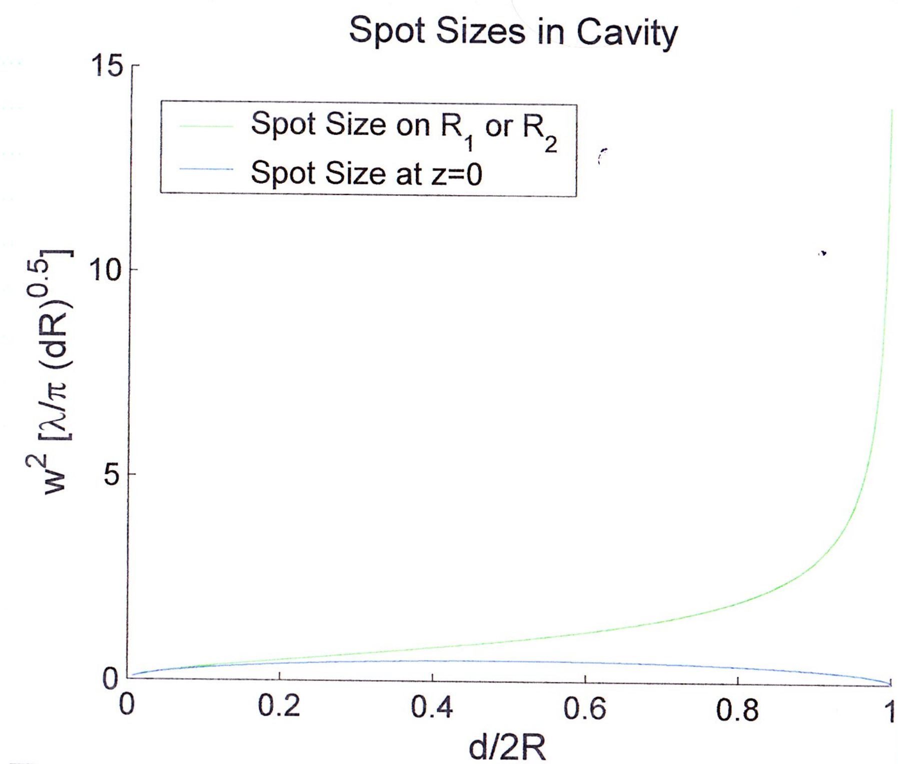
$$W^2(d/2) = \frac{\lambda_0}{\pi}\,\frac{\sqrt{dR/2}}{\sqrt{1 - d/2R}}$$ at ~~the~~ either mirror R $\pm d/2$

@ $z=0$

$$W^2(0) = \frac{\lambda_0}{\pi} z_0 = \frac{\lambda_0}{\pi}\sqrt{\frac{dR}{2}}\sqrt{1 - \frac{d}{2R}}$$

let $x = d/2R \qquad \sqrt{dR/2} = R\sqrt{x}$

$$W^2(0) = \frac{\lambda_0 R}{\pi}\sqrt{x}\sqrt{1-x} \qquad W^2(d/2) = \frac{\lambda_0 R}{\pi}\,\frac{\sqrt{x}}{\sqrt{1-x}}$$




```
function Lasers_Ch5_Pr4
textsize = 10;titlesize = 14;axissize = 12;
nx=200;
x = zeros(nx);for i = 1:nx;x(i) = i/nx;end;
% at mirror R1 or R2 at d/2
w2 = sqrt(x)./sqrt(1-x);
% at center z=0
w2_0 = sqrt(x).*sqrt(1-x);
axes('fontsize',axissize,'position',[.15 .19 .81 .7]);
ifigure = 1;figure(ifigure);cla;
title(['Spot Sizes in Cavity'],'fontsize',titlesize);
ylabel('w^2 [\lambda/\pi (dR)^0^.^5]','fontsize',titlesize);
xlabel('d/2R','fontsize',titlesize);hold on;
h1=plot(x,w2,'color',[0 .7 0]);
h1=plot(x,w2_0,'b');
h=legend(['Spot Size on R_1 or R_2';'Spot Size at z=0
lines = findobj(get(h,'children'),'type','line');
set(lines(1),'color',[0 .7  0]);
set(lines(2),'color',[0  0 1]);
```

5.4b Use the expansion law for a Gaussian beam to find the spot sizes on the mirrors.

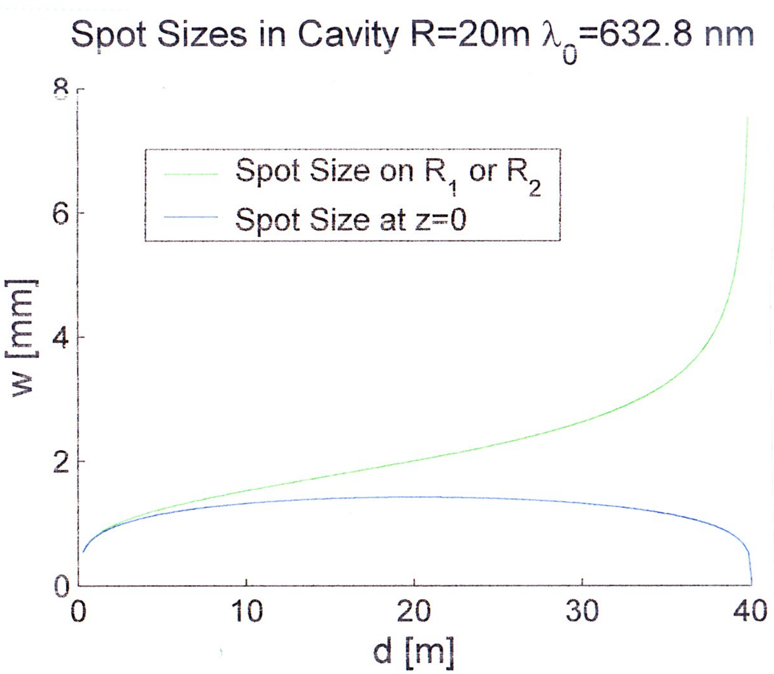
$$W(z>>z_0) = \frac{W_0 z}{z_0} = \frac{\lambda_0 z}{\pi n W_0} =$$

on mirror $z = d/2$

$$w = \frac{\lambda_0 d}{2\pi n W_0}$$

$$w(d/2) = \left(\frac{\lambda_0}{\pi}\frac{\sqrt{dR/2}}{\sqrt{1-d/2R}}\right)^{1/2}$$

using example 5.4 parameters



Source code →

5.4c If $R_1 = R_2$, find the distance that maximizes the mode volume.

$$E_0^2 V = \int_0^d \int_{-\infty}^{\infty}\int_{-\infty}^{+\infty} E(x,y,z)E^*(x,y,z)\,dx\,dy\,dz$$

$$E_0^2 V_{m,p} = E_0^2 \int_0^d \frac{W_0^2}{W^2(z)} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H_m^2\left(\frac{\sqrt{2}x}{w}\right)e^{-2x^2/\omega^2} H_p^2\left(\frac{\sqrt{2}y}{\omega}\right)e^{-2y^2/\omega^2}\,dx\,dy\,dz$$

$$V_{m,p} = \int_0^d \frac{W_0^2}{2}dz \int_{-\infty}^{+\infty} H_m^2(u)e^{-u^2}du \int_{-\infty}^{\infty} H_p^2(u)e^{-u^2}du$$

$$V_{m,p} = \int_0^d \frac{W_0^2}{2}dz\,[2^m m!\pi^{1/2}][2^p p!\pi^{1/2}]$$

$$V_{m,p} = \frac{\pi W_0^2}{2} d\; m!p!\,2^{m+p} = A W_0^2 d$$

$$W_0^2 = \frac{\lambda_0}{\pi}\sqrt{\frac{dR}{2}}\sqrt{1-\frac{d}{2R}}$$

$$V_{m,p} = A'\sqrt{\frac{dR}{2}}\left(\sqrt{1-\frac{d}{2R}}\right)d = A'\left[\left(\frac{dR}{2}\right)(d^2)\left(1-\frac{d}{2R}\right)\right]^{1/2}$$

$$V_{m,p} = A'\left[\frac{R}{2}d^3 - \frac{1}{4}d^4\right]^{1/2} \qquad A' = \frac{\lambda_0}{\pi}\frac{\pi}{2}m!p!\,2^{m+p}$$

$$\left.\frac{\partial V}{\partial d}\right|_{d_{max}} = A'\tfrac{1}{2}\left[\frac{R}{2}d^3 - \frac{1}{4}d^4\right]^{-1/2}\left[\frac{3R}{2}d^2 - d^3\right]\Bigg|_{d_{max}} = 0$$

$$\boxed{d = \frac{3R}{2}}$$

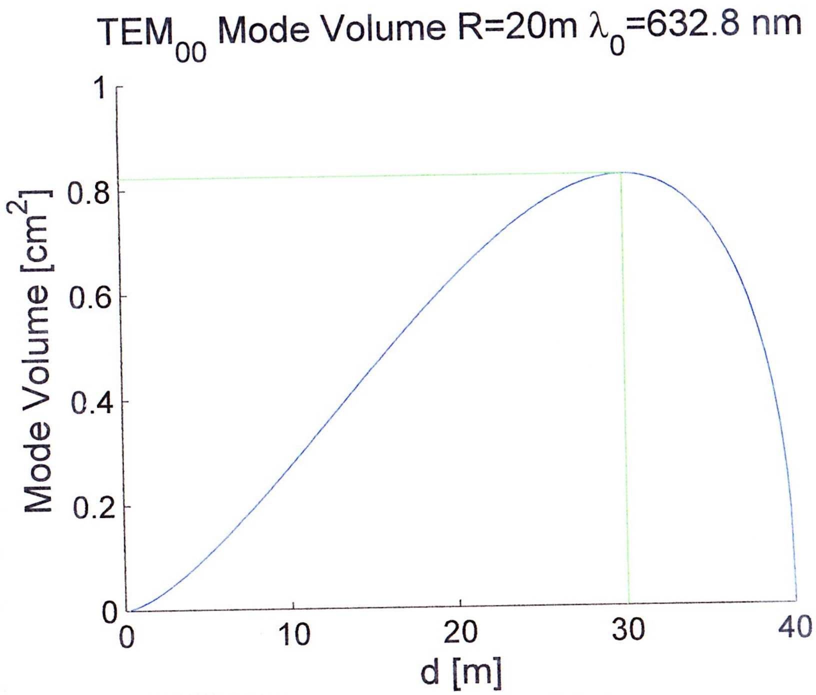
~~$\frac{\partial^2 V}{\partial d^2}$~~

Mode Volume

$$V_{m,p} = \frac{\lambda_0}{2} m!p!\,2^{m+p}\sqrt{\frac{R}{2}d^3 - \frac{1}{4}d^4} \quad \text{for } TEM_{m,p}$$

Cavity length for maximum volume

$$d_{max} = \frac{3R}{2} \qquad V_{m,p}\Big|_{max\,d} = \frac{\lambda_0}{2}m!p!\,2^{m+p}R^2\sqrt{\left(\frac{27}{1}-\frac{1}{2}\right)\left(\frac{1}{2^4}\right)}$$



C:\MATLAB6p5p2\work\Lasers_Ch5_Pr4b.m
May 3, 2009

Mode Volume

```
function Lasers_Ch5_Pr4b
textsize = 10;titlesize = 14;axissize = 12;
pi = 4.0*atan(1.0);
nx=200;d=0.5;R=20.0;L=632.8e-9;dmax=3.0*R/2.0;
d = zeros(nx);for i = 1:nx;d(i) = 2*R*i/(nx);end;
m = 0;p=0;
A = L*factorial(m)*factorial(p)*(2^m)*(2^p)/2;
V00 = A*sqrt((R/2)*(d.^3) - 0.25*(d.^4))*10000;
Vmax = A*sqrt((R/2)*(dmax^3) - 0.25*(dmax^4))*10000;
axes('fontsize',axissize,'position',[.15 .19 .81 .7]);
ifigure = 1;figure(ifigure);cla;
title(['TEM_0_0 Mode Volume R=20m \lambda_0=632.8 nm'], ...
      'fontsize',titlesize);
ylabel('Mode Volume [cm^2]','fontsize',titlesize);
xlabel('d [m]','fontsize',titlesize);hold on;
h1=plot(d,V00,'color',[0 0 .7]);
h2=plot([dmax dmax],[0 Vmax],'color',[0 .7 0]);
h2=plot([0 dmax],[Vmax Vmax],'color',[0 .7 0]);
```

Plot spot size at R_1 (R_2) and z=0 at center

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May 3, 2009

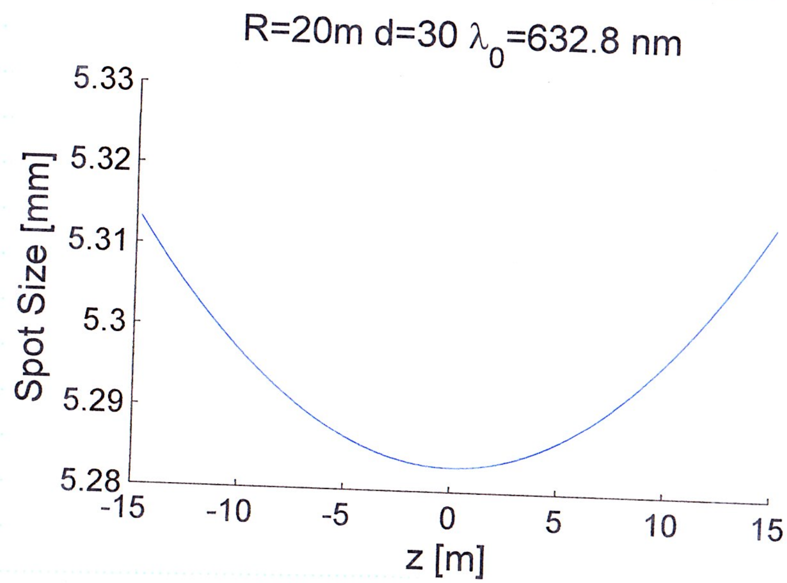
```
function Lasers_Ch5_Pr4a
% plot the spot size at R and z=0 for Problem 5.4
textsize = 10;titlesize = 14;axissize = 12;
pi = 4.0*atan(1.0);
nx=200;d=0.5;R=20.0;L=632.8e-9;A=L*sqrt(R/2.0)/pi;
R2=2.0*R;
d = zeros(nx);for i = 1:nx;d(i) = 2*R*i/(nx);end;
% at mirror R1 or R2 at d/2
w2 = 1000.0*sqrt(A*sqrt(d)./sqrt(1-d./R2));
% at center z=0
w2_0 = 1000.0*sqrt(A*sqrt(d).*sqrt(1-d./R2));
axes('fontsize',axissize,'position',[.15 .19 .81 .7]);
ifigure = 1;figure(ifigure);cla;
title(['Spot Sizes in Cavity R=20m \lambda_0=632.8 nm'], ...
    'fontsize',titlesize);
ylabel('w [mm]','fontsize',titlesize);
xlabel('d [m]','fontsize',titlesize);hold on;
h1=plot(d,w2,'color',[0 .7 0]);
h1=plot(d,w2_0,'b');
h=legend(['Spot Size on R_1 or R_2';'Spot Size at z=0        ']);
lines = findobj(get(h,'children'),'type','line');
set(lines(1),'color',[0 .7 0]);
set(lines(2),'color',[0 0 1]);
```

For cavity with Volume Maximized : Spot size along z

At maximum volume $d = {}^{3R}/_{2}$ $R_1 = R_2$

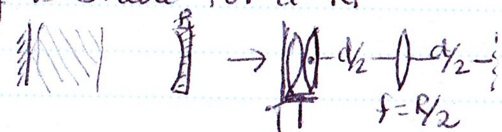
$$z_0 = \sqrt{\frac{dR}{2}}\sqrt{1 - {}^{d}/_{2R}} = \sqrt{\frac{3R^2}{4}}\sqrt{1-{}^{3}/_{4}} = 4\sqrt{3}R = \frac{\pi W_0^2}{\lambda_0}$$

$$W^2(z) = \frac{\lambda_0 z_0}{\pi}\left[1 + ({}^{z}/_{z_0})^2\right] = \frac{\lambda_0 4\sqrt{3}R}{\pi}\left[1 + \frac{z}{48R^2}\right]$$




```
function Lasers_Ch5_Pr4c
textsize = 10;titlesize = 14;axissize = 12;
pi = 4.0*atan(1.0);
nx=200;d=0.5;R=20.0;L=632.8e-9;dmax=3.0*R/2.0;
z = zeros(nx);for i = 1:nx;z(i) = dmax*i/(nx)-dmax/2;end;
z0 = 4.0*sqrt(3)*R;z02=48*R^2;
w = 1000.0*sqrt((L*z0/pi)*(1+(z.^2)/z02));
axes('fontsize',axissize,'position',[.15 .19 .81 .7]);
ifigure = 1;figure(ifigure);cla;
title(['R=20m d=30 \lambda_0=632.8 nm'], ...
    'fontsize',titlesize);
ylabel('Spot Size [mm]','fontsize',titlesize);
xlabel('z [m]','fontsize',titlesize);hold on;
h1=plot(z,w,'color',[0 0 .7]);
```

5.5 a) If $d/2$ of the space adjacent to the flat mirror of (5.1) were filled with a negative gas lens (2.12.12) show that the cavity is stable for $d = R$.



$$T = \begin{bmatrix} \cosh{}^{2d}/_{L} & L\sinh{}^{2d}/_{L} \\ {}^{1}/_{L}\sinh{}^{2d}/_{L} & \cosh{}^{2d}/_{L} \end{bmatrix}\begin{bmatrix} 1 & d/2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}\begin{bmatrix} 1 & d/2 \\ 0 & 1 \end{bmatrix}$$

or

$$T = \begin{bmatrix} 1 & d/2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}\begin{bmatrix} 1 & d/2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \cosh{}^{d}/_{L} & L\sinh{}^{d}/_{L} \\ {}^{1}/_{L}\sinh{}^{d}/_{L} & \cosh{}^{d}/_{2} \end{bmatrix}\begin{bmatrix} \cosh{}^{d}/_{L} & L\sinh{}^{d}/_{L} \\ {}^{1}/_{L}\sinh{}^{d}/_{L} & \cosh{}^{d}/_{L} \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & d/2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}\begin{bmatrix} 1 & d/2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \cosh^2\frac{d}{L} + \sin^2\frac{d}{L} & 2L\cosh{}^{d}/_{L}\sin{}^{d}/_{L} \\ \frac{2}{L}\cosh{}^{d}/_{L}\sinh{}^{d}/_{L} & \sin^2{}^{d}/_{L} + \cosh^2{}^{d}/_{L} \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & d/2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}\begin{bmatrix} 1 & d/2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \cosh(2d/L) & L\sinh(2d/L) \\ \frac{1}{L}\sinh(2d/L) & \cosh(2d/L) \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & d/2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}\left[\cosh\left(\tfrac{2d}{L}\right) + \tfrac{d}{2L}\sinh\left(\tfrac{2d}{L}\right)\right.$$

$$T = \begin{bmatrix} \cos{}^{2d}/_{L} & L\sinh{}^{2d}/_{L} \\ {}^{1}/_{L}\sinh{}^{2d}/_{L} & \cosh{}^{2d}/_{L} \end{bmatrix}\begin{bmatrix} 1 & d/2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & d/2 \\ -2/R & -({}^{2}/_{R})({}^{d}/_{2}) + 1 \end{bmatrix}$$

$$T = \begin{bmatrix} \cosh{}^{d}/_{L} & L\sinh{}^{d}/_{L} \\ {}^{1}/_{L}\sinh{}^{d}/_{L} & \cosh{}^{d}/_{L} \end{bmatrix}\begin{bmatrix} 1 - {}^{d}/_{R} & d - {}^{d^2}/_{2R} \\ -{}^{2}/_{R} & -{}^{d}/_{R} + 1 \end{bmatrix}$$

$$T = \begin{bmatrix} (1-\frac{d}{R})\cosh{}^{d}/_{L} - {}^{2L}/_{R}\sinh{}^{d}/_{L} & (d - {}^{d^2}/_{2R})\cosh{}^{d}/_{L} + (1-{}^{d}/_{R})L\sinh{}^{d}/_{L} \\ (1-{}^{d}/_{R})\frac{1}{L}\sinh{}^{d}/_{L} - {}^{2}/_{R}\cosh{}^{d}/_{L} & (d-{}^{d^2}/_{2R})\frac{1}{L}\sinh{}^{d}/_{L} + (1-{}^{d}/_{R})\cosh{}^{d}/_{L} \end{bmatrix}$$

if $d = R$

$$T = \begin{bmatrix} -{}^{2L}/_{d}\sinh{}^{d}/_{L} & {}^{1}/_{d}\cosh{}^{d}/_{L} \\ -{}^{2}/_{d}\cosh{}^{d}/_{L} & {}^{1}/_{dL}\sinh{}^{d}/_{L} \end{bmatrix}$$

Stability criteria

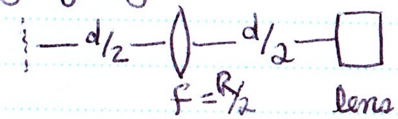
$$S = \frac{A+D+2}{4}$$

$$S = \frac{\left(-2L/\cancel{d} + 1/dL\right)\left(\sinh d/l\right) + 2}{4}$$

if $d=R$ then $L=d$ also

$$S = \frac{\left(-2 + 1/d^2\right)\sinh 1 + 2}{4} \qquad \sinh(1) = 1.17520$$

trying again start cell right after gas lens



lens of effective length = d $(d/2 + d/2)$

$$T_{gc} = \begin{bmatrix} \cosh(d/L) & L\sinh(d/L) \\ \frac{1}{L}\sin(d/L) & \cosh(d/L) \end{bmatrix} \quad T_m = \begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix} \quad T_{space} = \begin{bmatrix} 1 & d/2 \\ 0 & 1 \end{bmatrix}$$

$$T = T_{gascell} T_{space} T_{mirror} T_{space} \qquad d=R$$

$$T = \begin{bmatrix} \cosh d/L & L\sinh d/L \\ 1/L \sinh d/L & \cosh d/L \end{bmatrix}\begin{bmatrix} 1 & d/2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2/d & 1 \end{bmatrix}\begin{bmatrix} 1 & d/2 \\ 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} \cosh d/L & L\sinh d/L \\ 1/L \sinh d/L & \cosh d/L \end{bmatrix}\begin{bmatrix} 1 & d/2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & d/2 \\ -2/d & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} \cosh d/L & L\sinh d/L \\ 1/L \sinh d/L & \cosh d/L \end{bmatrix}\begin{bmatrix} 0 & d/2 \\ -2/d & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} (-2L/d)\sinh d/L & (d/2)\cosh d/L \\ (-2/d)\cosh d/L & (d/2L)\sinh d/L \end{bmatrix}$$

$$S = \frac{A+D+2}{4} = \frac{\sinh d/L \left(\frac{d}{2L} - \frac{2L}{d}\right)}{4} + \frac{1}{2}$$

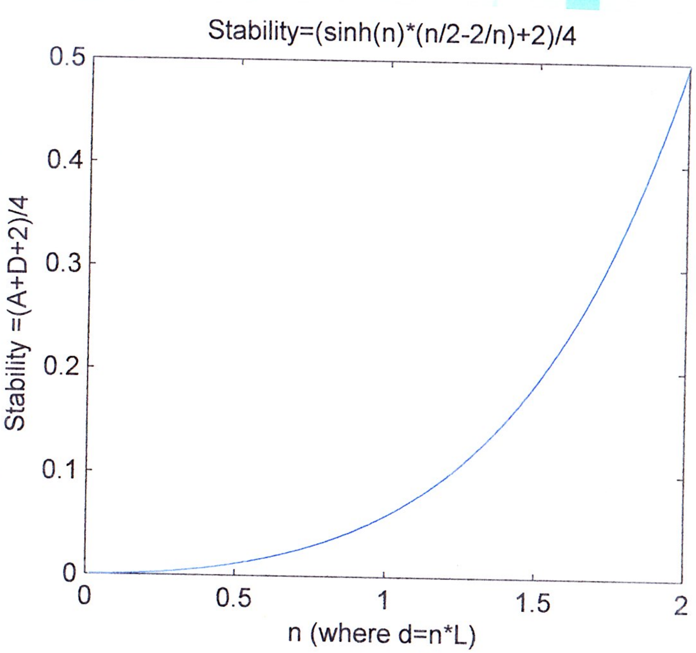
let $d = nL$ — stable for $0 \le n \le 2.3993$ gives $0 \le S \le 1$ found numerically

$$S = \frac{\sinh(n)\left(\frac{n}{2} - \frac{2}{n}\right)}{4} + \frac{1}{2}$$

$$S = 0 = \sinh(n)\left(\frac{n}{2} - \frac{2}{n}\right)2 + 1 = \left(n - \frac{4}{n}\right)\sinh n + 1 = 0$$

invalid when $n=0$ $\rightarrow$ $\sinh n = \frac{1}{4/n - n} = \frac{n}{4-n^2} \leftarrow$ $n>0$, $n<2$

```
function Lasers_Ch5_Pr5
x = 0:0.01:2.0;
s=((x./2-2./x).*sinh(x)+2)/4;
plot(x,s);
xlabel('n (where d=n*L)');
ylabel('Stability =(A+D+2)/4');
title('Stability=(sinh(n)*(n/2-2/n)+2)/4');
```



(b) Find the spot size

$$w^2 = \frac{\lambda_0}{\pi n} \frac{B}{\left[1 + \left(\frac{A+D}{2}\right)^2\right]^{1/2}}$$

$$w^2 = \frac{\lambda_0}{\pi n} \frac{\frac{d}{2}\cosh\frac{d}{L}}{\left[1 + \left(\sinh^2\frac{d}{L}\right)\left(\frac{1}{4}\right)\left(\frac{d}{2L} - \frac{2L}{d}\right)^2\right]^{1/2}}$$